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Trained stochastic model of biological neural network used in image processing task

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ABSTRACT

In this paper we are presenting some new approaches to the biological model of neural network, which is strictly based on Hodgkin–Huxley types of models. The first aspect was to introduce stochasticity into a model of dendritic structure of neuron already proposed in Hodgkin and Huxley (1952) by using Markov kinetic schemes (Destexhe et al., 1994). Second thing was to bring into this model a training algorithm which is based on the descent gradient method. Subsequently the trained neural network is supposed to solve a problem of noise removal from a given image.

This study is supposed to underline potential of biologically realistic models of neural network, which – with a bit of invention – can be used like conventional artificial neural networks.

1. Introduction

Nowadays, it is possible to perceive the increasing number of scientists that are interested in modeling behavior of biological neuron and neural network [3]. This interest is related not only to designing new models but also to using them in real-life applications, just like conventionally used artificial neural networks. When it comes to a practical application an important question is how such neural networks should be realized. The most popular are various software implementations, as they are the most flexible. However, recently hardware realizations are more and more frequently considered in the context of emerging technologies such as portable medical health care systems. Hardware implementations include field programmable gate array (FPGA) realizations [4] but also application specific integrated circuits (ASICs) [5–10].

In this paper we are trying to combine few aspects of the models that are mentioned above, namely: reliable description of processes that take place in the neurons or neural networks, which is done by introducing stochasticity into these models; to force the training procedure on these models; and finally to use trained neural network in real problems – in this case in image processing.

There are a lot of models that reflect the spiking nature of the neuron and at the same time are used in applications. One of the commonly known model is the Izhikevich model presented in [11], which is widely used in applications in many fields, e.g. great examples are shown in [12,13], where the Izhikevich model was used in the process of control of the arm of humanoid robot iCub. This shows only minor possibilities of this kind of models.

In this paper we present a collection of new approaches and solutions for biological model of neural network. We combined the model of biological neural network presented in [1] with the idea of Markov kinetic schemes for description of ion movement through the membrane of the neuron, which allows to show the interaction between stochastic discrete

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elements, which are the ion gates – the components of the neuron's membrane [14]. The next step described in this paper is to introduce the training algorithm into the newly derived kinetic model of neural network. This procedure leads straight to the search of potential applications, in this case we present a problem of image processing.

The paper is organized as follows. In Section 2 we provide an overall description of Hodgkin–Huxley model of neuron and its kinetic extension. The kinetic model is presented in two versions: deterministic and stochastic and it is based on Markov kinetic schemes. Section 3 is focused on developing the kinetic model of dendritic structure of the neuron (further referred as neural network) which is also based on the Hodgkin–Huxley model presented in [1]. We are shortly describing the process of the discretization of the equations of this model, which is based on the idea presented in [15]. In Section 6 the problem of minimization is shown, while in Section 7 – the way of solving it, which is based on two methods: of descent gradient presented in Section 4 and of Lagrange multipliers presented in Section 5. Finally, in Section 8 we are focusing on results of the application in the image processing field, while in Section 9 we give a summary.

2. Hodgkin–Huxley model and its kinetic extension

Hodgkin–Huxley model has been an inspiration for many subsequent researchers since it was designed and published in 1952 [1]. In this model it is assumed that the dynamics of the potential on the neuron's membrane can be written in the following form [1,3,16,17]:

$$C\frac{dV}{dt} = I - g_{Na}m^3h(V - V_{Na}) + g_K n^4(V - V_K) + g_L(V - V_L) \tag{1}$$

where V represents the potential, I – input current (given from the external source), while remaining elements of the sum are the components of the ion current which is related to the flow of the certain types of ions (sodium, potassium and chloride) through the membrane. The elements $g_{Na}m^3h, g_K n^4$ and g_L have the dimension of ion conductance and are expressing the transmission of the membrane. Parameters m, n and h are dimensionless variables with values in range from 0 to 1. These variables are describing the nature of the membrane. In the Hodgkin–Huxley model it is assumed that the membrane is built from channels which consist from small gates. These small components are supposed to control the movement of ions through the membrane. In this case we assume that each gate can be in one of two possible states: permissive or non-permissive, which is expressed with the following set of differential equations [1,3,16,17]:

$$\begin{aligned} \frac{dn}{dt} &= \alpha_n(V)\cdot(1-n) - \beta_n(V)\cdot n \\ \frac{dm}{dt} &= \alpha_m(V)\cdot(1-m) - \beta_m(V)\cdot m \\ \frac{dh}{dt} &= \alpha_h(V)\cdot(1-h) - \beta_h(V)\cdot h \end{aligned} \tag{2}$$

where $\alpha(V)$ and $\beta(V)$ are so called transfer functions and are shown in Table 1. Additionally in Table 2 we are showing the values of potentials V_i and conductances g_i used in the model, while $C = 1$ $[\mu F/cm^2]$.

The kinetic extension of the Hodgkin–Huxley model, which can be considered in deterministic and stochastic version, is based on the idea that a single gate can be in more than just two states [18]. In this model it is assumed that potassium gates can be in five states, where one is permissive and the remaining ones are non-permissive and it is expressed with the Markov kinetic scheme showed in (3). Notation $[n_i]$ refers to the number i of permissive gates related to the potassium ions. In the kinetic model four permissive gates are necessary, so the flow of potassium ions was possible, hence in the main equation we use $[n_4]\cdot g_K$ instead of $g_K n^4$ [18].

$$n_0 \underset{\beta_n}{\overset{4\alpha_n}{\rightleftharpoons}} n_1 \underset{2\beta_n}{\overset{3\alpha_n}{\rightleftharpoons}} n_3 \underset{3\beta_n}{\overset{2\alpha_n}{\rightleftharpoons}} n_0 \underset{4\beta_n}{\overset{\alpha_n}{\rightleftharpoons}} n_4 \tag{3}$$

$$\begin{array}{ccccccc} m_0h_0 & \underset{\beta_m}{\overset{3\alpha_m}{\rightleftharpoons}} & m_1h_0 & \underset{2\beta_m}{\overset{2\alpha_m}{\rightleftharpoons}} & m_2h_0 & \underset{3\beta_m}{\overset{\alpha_m}{\rightleftharpoons}} & m_3h_0 \\ \alpha_h \upharpoonleft\downharpoonright \beta_h & & \alpha_h \upharpoonleft\downharpoonright \beta_h & & \alpha_h \upharpoonleft\downharpoonright \beta_h & & \alpha_h \upharpoonleft\downharpoonright \beta_h \\ m_0h_1 & \underset{\beta_m}{\overset{3\alpha_m}{\rightleftharpoons}} & m_1h_1 & \underset{2\beta_m}{\overset{2\alpha_m}{\rightleftharpoons}} & m_2h_1 & \underset{3\beta_m}{\overset{\alpha_m}{\rightleftharpoons}} & m_3h_1 \end{array} \tag{4}$$

Table 1
$\alpha_i(V)$ and $\beta_i(V)$ functions [1].

i	$\alpha_i(V)$	$\beta_i(V)$
n	$\frac{0.01\cdot(10-V)}{\exp\left(\frac{10-V}{10}\right)-1}$	$0.125\cdot\exp\left(-\frac{V}{80}\right)$
m	$\frac{0.1\cdot(25-V)}{\exp\left(\frac{25-V}{10}\right)-1}$	$4\cdot\exp\left(-\frac{V}{18}\right)$
h	$0.07\cdot\exp\left(-\frac{V}{20}\right)$	$\frac{1}{\exp\left(\frac{30-V}{10}\right)+1}$