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Form finding and analysis of inflatable dams using dynamic relaxation

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ABSTRACT

Inflatable dams are flexible membrane structures inflated by air and/or water. Due to their ease of construction, rapid deployability and low cost, these systems have great potential for hazard mitigation applications in the context of global warming. However, designing inflatable dams is a challenging task as the dam's initial equilibrium shape has to be determined by either experimental or numerical form-finding methods. Furthermore, the dam's shape and the applied loading are coupled since changes in the form of the structure induce also changes in the loading profile. In this paper, dynamic relaxation, a well-established form-finding and analysis technique, is employed for the cross-sectional analysis of inflatable dams. Using this technique and the proposed extensions, the structural behavior of inflatable dams can be analyzed under constant and varying internal pressure as well as different loading and support conditions. The results are in agreement with published results in literature. Therefore, the presented method provides an alternative computationally advantageous tool for the design of inflatable dams.

1. Introduction

In the last three decades, the number and intensity of tropical storms hitting the eastern coast of the United States of America has risen. From 1980 to 2011, tropical cyclones, severe local storms and non-tropical floods encountered for 67% of all natural disasters causing more than $1 billion (USD) in damage [1]. Most of the damage caused by these extreme events was due to storm surge flooding in coastal communities. Storms such as hurricane Sandy, which devastated New York and New Jersey in 2012 and caused an estimated $65 billion in damage and claiming over 150 lives [2], have emphasized the need for novel flood mitigation systems in urban areas. Storm surge flooding can be reduced with the use of coastal storm surge barriers, usually taking the form of earthen or concrete structures. However, such structures have a tendency to permanently affect the environment. Moreover, they impede residential water views, thus negatively affecting property values in the area they are being constructed [3]. For this reason, these traditional storm surge barriers are often met with resistance, making temporary/deployable systems such as inflatable membrane dams an attractive alternative [4].

Inflatable dams are flexible membrane structures inflated by air and/or water that have been commonly used as a substitute for traditional earth dams [5]. They have also been employed to increase the height or reservoir capacity of existing dams, to mitigate chemical spills and flow rates in rivers, as well as for waste water management [6]. Their most attractive feature is their adaptability as their shape can be varied based on their internal pressure. Changing pressure and, in turn, shape can thus be used to either choke, or encourage the flow of water based on environmental conditions [5].

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Predicting the equilibrium shape of inflatable dams is challenging. Therefore, analytical solutions are usually based on material idealizations and shape approximations. Anwar [7] approximated the hydro-static equilibrium profile based on the equation of a circle, for the free downstream membrane portion, and using elliptic integrals, for the loaded upstream membrane. Binnie [8] developed a closed form solution for water inflated dams accounting for physical restrictions of the base and perimeter lengths. However, both solutions are based on an idealized weightless, inextensible membrane as well as linked to specific load cases.

To take into account material properties and multiple load cases, numerical methods have been proposed. Harrison [6] presented an iterative numerical analysis method based on a finite element model that can handle hydro-static loading from both sides and includes the self-weight of the membrane. In his method, the equilibrium profile of the dam is obtained based on the local equilibrium of the elements composing the membrane. Harrison's method was adapted by Alhamati et al. [9] who also validated it experimentally. Parbery [10,11] proposed a numerical solution for inflatable dams based on the differential equations of equilibrium and investigated factors affecting the membrane dam inflated by air pressure. However, like Harrison's method, Parbery's method is limited to profiles with two distinct anchorage points and constant internal pressure. Therefore, this paper focuses on providing a more general numerical tool for the design of inflatable dams.

Inflatable dams are force-modeled structures: their equilibrium shape must be determined by either experimental or numerical form-finding methods. Furthermore, the dam's shape and the applied loading are coupled since changes in the form also induce changes in the loading. Due to the loading-shape interaction, the design of inflatable dams is more challenging than conventional force-modeled structural systems. In this paper, an iterative algorithm based on dynamic relaxation is proposed for the form-finding and analysis of inflatable dams. Dynamic relaxation is an established, explicit, numerical form-finding and analysis method [12]. In Section 2, the basic dynamic relaxation scheme is presented. In Section 3, the proposed dynamic relaxation algorithm and its related extensions for the cross-sectional analysis of inflatable dams are discussed. Finally, conclusions are presented in Section 4.

2. Dynamic relaxation: the basic scheme

Dynamic relaxation is a well-established form-finding and analysis method that has existed for more than 40 years [13]. In dynamic relaxation, structures are modeled as a mesh of elements connected with nodes. Nodes are assigned with mass, loads and damping. The method traces the motion of each unconstrained node of the structure for small time increments until, due to artificial damping, the structure reaches a static equilibrium. Dynamic relaxation is thus based on Newton's second law for each node of the structure:

$$F_{ext} - F_{int} = M\dot{v} + Dv \tag{1}$$

where F_{ext} and F_{int} are the external and internal forces respectively. $\dot{v}$ and v are the nodal acceleration and the velocity respectively. M corresponds to the nodal mass and D to damping with both parameters being fictitious and optimized for the stability and convergence of the method [14–17].

Dynamic relaxation has been employed for the static analysis of truss structures [18,19], cable structures [20–22], tensegrity structures [23,24], prestressed net structures and membranes [25–27], reciprocal frames [28], spline structures [29] as well as for dynamic analyzes [30,31]. In this paper, dynamic relaxation is employed for the cross-sectional analysis of inflatable membrane dams. Therefore, only tensile elements are employed. Furthermore, "kinetic" damping is used to obtain static equilibrium. The motion of the structure is traced and when a local peak in the total kinetic energy of the system is detected, all velocity components are set to zero. The process is then restarted from the current geometry and repeated until the energy of all modes of vibration has been dissipated and a static equilibrium has been achieved. Hence, the damping term Dv of Eq. (1) is abandoned.

Expressing accelerations in a finite difference form provides the velocity and the updated geometry for each node:

$$v^{t+\Delta t/2} = v^{t-\Delta t/2} + [(F_{ext} - F_{int})\Delta t]/M \tag{2}$$

$$x^{t+\Delta t} = x^{t} + v^{t+\Delta t/2}\Delta t \tag{3}$$

where $v^{t+\Delta t/2}$ and $v^{t-\Delta t/2}$ are the nodal velocities at times $t+\Delta t/2$ and $t-\Delta t/2$ respectively. $x^{t+\Delta t}$ is the nodal position at time $t+\Delta t$ and Δt is the time step applied. Consequently, for every time step a new geometry is obtained. Based on the new geometry, new internal forces can be estimated using Hooke's law and thus starting over. Convergence is reached when the term $(F_{ext} - F_{int})$ in Eq. (1) is sufficiently small.

3. Dynamic relaxation for the cross-sectional analysis of inflatable dam analysis

3.1. Force update

The shape of an inflatable membrane dam and the applied hydrostatic loading are coupled since changes in the form can also induce variations in the loading and vice versa. Therefore, this first extension focuses on the implementation of the loading-shape interaction. In this paper, the applied loading includes both the hydrostatic force and the internal pressure