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Nonlocal impulsive fractional differential inclusions with fractional sectorial operators on Banach spaces ☆

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ABSTRACT

This paper investigates existence of *PC*-mild solutions of impulsive fractional differential inclusions with nonlocal conditions when the linear part is a fractional sectorial operators like in Bajlekova (2001) [1] on Banach spaces. We derive two existence results of *PC*-mild solutions when the values of the semilinear term F is convex as well as another existence result when its values are nonconvex. Further, the compactness of the set of solutions is characterized.

1. Introduction

Fractional differential equations and fractional differential inclusions arise in many engineering and scientific disciplines as the mathematical modeling of systems and processes in the fields of physics, chemistry, aerodynamics, electrodynamics of a complex medium, polymer rheology, etc., involves derivatives of fractional order. Fractional differential equations also serve as an excellent tool for the description of hereditary properties of various materials and processes. For some applications of fractional differential equations, one can see [26,28,33–36,40] and the references therein. It seems that El-Sayed and Ibrahim [22] initiated the study of fractional multivalued differential inclusions. Recently, some basic theory for initial value problems for fractional differential equations and inclusions was discussed by Kilbas et al. [33], Lakshimkantham et al. [34], Miller et al. [36], Podlubny [40] and the papers [1–3,5,12,18,23,27,35,42–49] and the references therein.

The theory of impulsive differential equations and impulsive differential inclusions has been an object interest because of its wide applications in physics, biology, engineering, medical fields, industry and technology. The reason for this applicability arises from the fact that impulsive differential problems are an appropriate model for describing process which at certain moments change their state rapidly and which cannot be described using the classical differential problems. For some of these applications we refer to [6,10]. During the last ten years, impulsive differential inclusions with different conditions

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have intensely student by many mathematicians. At present, the foundations of the general theory of impulsive differential equations and inclusions are already laid, and many of them are investigated in details in the book of Benchohra et al. [13].

Moreover, a strong motivation for investigating the nonlocal Cauchy problems, which is a generalization for the classical Cauchy problems with initial condition, comes from physical problems. For example, it used to determine the unknown physical parameters in some inverse heat condition problems. For the applications of nonlocal conditions problems we refer to [11,25]. In the few past years, several papers have been devoted to study the existence of solutions for differential equations or differential inclusions with nonlocal conditions [7]. For impulsive differential equation or inclusions with nonlocal conditions of order one we refer to [16,24]. For impulsive differential equation or inclusions of fractional order we refer to [4,21,41,45] and the references therein.

In this paper we are concerned with the existence of mild solutions to the following nonlocal impulsive fractional differential inclusions of the type

$$\begin{cases} {}^cD^{\alpha}x(t) \in Ax(t) + F(t, x(t)), \alpha \in (0,1), \quad \text{a.e. on} \quad J - \{t_1, t_2, \dots, t_m\}, \\ x(0) = x_0 - g(x), \\ x(t_i^+) = x(t_i) + I_i(x(t_i)), \quad i = 1, 2, \dots, m, \end{cases} \tag{1}$$

where $J := [0, b]$ with $b > 0$ is fixed, ${}^cD^{\alpha}$ is the Caputo fractional derivative of the order $\alpha \in (0,1)$ with the lower limit zero, A is a fractional sectorial operator like in [1] defined on a separable Banach space E, $F : J \times E \to 2^E - \{\phi\}$ is a multifunction, $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = b$, $I_i : E \to E$ $(i = 1, 2, \dots, m)$ are impulsive functions which characterize the jump of the solutions at impulse points t_i, $g : PC(J, E) \longrightarrow E$, is a nonlinear function related to the nonlocal condition at the origin and $x(t_i^+)$, $x(t_i^-)$ are the right and left limits of x at the point t_i respectively and $PC(J, E)$ will be define later.

Concerning with the main problem (1), we have to study the following impulsive fractional evolution equations with nonlocal conditions:

$$\begin{cases} {}^cD^{\alpha}x(t) = Ax(t) + f(t, x(t)), \alpha \in (0,1), \quad \text{a.e. on} \quad J - \{t_1, t_2, \dots, t_m\}, \\ x(0) = x_0 - g(x), \\ x(t_i^+) = x(t_i) + I_i(x(t_i)), \quad i = 1, 2, \dots, m. \end{cases} \tag{2}$$

In [44], Wang et al. introduced a new concept of mild solutions for (2) and derived existence and uniqueness results concerning the PC-mild solutions for (2) when f is a Lipschitz single-valued function or continuous and maps bounded sets into bounded sets and A is the infinitesimal generator of a compact semigroup $\{T(t),\ t \geqslant 0\}$.

After reviewing the previous research on the fractional evolution equations, we find that the operator in the linear part is the infinitesimal generator of a strongly continuous semigroup, an analytic semigroup, or compact semigroup, or a Hille–Yosida operator, much less is known about the fractional evolution (differential) inclusions with sectorial or almost sectorial operators.

In order to do a comparison between our obtained results in this paper and the known recent results in the same topic, we would like to mention that, recently, the study of evolution equations involving sectorial or almost sectorial operators has been investigated to a large extent. For example, Shu et al. [42] introduced a new concept of mild solutions for impulsive fractional evolution equations and derived existence results concerning the mild solutions for (2) when F is a completely continuous single-valued function, $g = 0$ and A is a sectorial operator such that the operators families $\{S_{\alpha}(t),\ t \geqslant 0\}$ and $\{T_{\alpha}(t),\ t \geqslant 0\}$ are compact. We will explain in Remark 2.21 why does the definition given in [42] not suitable in some sense. So, we will give another definition for PC-mild solutions for (1) based on the definition given by Wang et al. [44]. Periago and Straub [39] gave a functional calculus for almost sectorial operator, and using the semigroup of growth $1 + \gamma$ which is defined by this functional calculus, obtained the existence and uniqueness for Cauchy problems of abstract evolution equations involving almost sectorial operator, that is by constructing an evolution process of growth $1 + \gamma$. More recently, Wang et al. [46] considered abstract fractional Cauchy problem when F is a single valued, $g = 0$ and A is an almost sectorial operator whose resolvent satisfies the estimate of growth $-\gamma$ $(-1 < \gamma < 0)$ in a sector of the complex plane. Agarwal et al. [3] proved an existence result for (1) without impulses and when A is a sectorial operator and the dimension of E is finite. They studied the dimension of the set of mild solutions. Ouahab [38,11] proved a version of Fillippov's Theorem for (1) without impulse, $g = 0$ and A is an almost sectorial operator. The study of differential equations or evolution equations in which the linear part is the infinitesimal generator of C_0-semigroup has been investigated by many authors. Cardinali and Rubbioni [16] proved the existence of mild solutions to (1) when $\alpha = 1$ and the multivalued function F satisfies the lower Scorza-Dragoni property and $\{A(t),\ t \geqslant 0\}$ is a family of linear operator, generating a strongly continuous evolution operators. Henderson and Ouahab [29] considered the problem (1) when $A = 0$, and Zhou et al. [47,48] introduced a suitable definition of mild solution for (1) based on Laplace transformation and probability density functions for (1) without impulses when A is the infinitesimal generator of C_0-semigroup, F is single-valued function. Very recently, Wang and Ibrahim [45] proved existence and controllability results for (1) when A is the infinitesimal generator of C_0-semigroup and $\{T(t),\ t > 0\}$ is strongly equicontinuous C_0-semigroup. In addition, Ibrahim et al. [27] proved the existence of mild solutions to the problem (1) when the multivalued function F satisfies the lower Scorza-Dragoni property and A is the infinitesimal generator of a compact semigroup $\{T(t),\ t > 0\}$.