Applied Mathematics and Computation 257 (2015) 274–284

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Robust stability and stabilization of fractional-order linear systems with polytopic uncertainties

Liping Chen [a,*], Ranchao Wu [b], Yigang He [a], Lisheng Yin [a]

[a] School of Electrical Engineering and Automation, Hefei University of Technology, Hefei 230009, China
[b] School of Mathematics, Anhui University, Hefei 230039, China

ARTICLE INFO

Keywords:
Fractional-order linear system
Stability
Stabilization
Polytopic uncertainties

ABSTRACT

Robust stability and stabilization of fractional-order uncertain linear systems with order $\alpha : 0 < \alpha < 1$ and $1 \leqslant \alpha < 2$ are considered in the paper. A new polytopic type uncertain state-space model for fractional-order linear systems is addressed, which allows second-order uncertain parameters. The uncertainty in the fractional-order model appears in terms of a polytope of matrices. Some sufficient criteria for the robust asymptotical stable and stabilization for such fractional-order uncertain linear systems are derived. All the results are obtained in terms of linear matrix inequalities (LMIs). Numerical examples are presented to demonstrate the validity and feasibility of the obtained results.

1. Introduction

The fractional calculus dates from the 17th century, is a classical mathematical notion and a generalization of the ordinary differentiation and integration to arbitrary non-integer order (or, complex order). Compared with traditional integer-order models, the advantages of the fractional-order derivation lie in the following two aspects. First, the integer-order derivative indicates a variation or certain attribute at particular time for a physical or mechanical process, while fractional-order derivative is concerned with the whole time domain. Second, integer-order derivative describes the local properties of a certain position for a physical process, by contrast, fractional-order derivative is related to the whole space. Taking into account these facts, the fractional calculus plays an important role in the modeling of many phenomena in various fields of science and engineering, such as quantum mechanics, stochastic diffusion, molecular spectroscopy, viscoelastic dynamics, control theory, robotics, etc. The study on theory and application of fractional-order dynamics system become excitement of international academia [1–7].

With more and more physical problems are well characterized by fractional-order state equations, to analyze its stability is the first essential and fundamental issue to be dealt with. Very recently, the stability problem of fractional-order systems has been investigated both from an algebraic and an analytic point of view. Some interesting stability results have obtained including fractional-order linear time-invariant systems [8], fractional-order linear delayed systems [9], fractional-order nonlinear systems [10] and fractional-order impulsive systems [11], etc. In fact, real systems have many uncertain characteristics, for examples, system parameters always fluctuate within some scopes. Thus, the stability analysis of fractional-order systems with uncertainty is very important. Much considerable attentions have been devoted to the problem of stability analysis and control design for fractional-order linear time invariant (FO-LTI) interval uncertainties systems [12–20]. In [12], a robust

* Corresponding author.
E-mail address: lip_chenhut@126.com (L. Chen).

http://dx.doi.org/10.1016/j.amc.2014.12.103
0096-3003/

stability test procedure and Matlab code for uncertain FO-LTI systems with interval coefficients described in state space form were presented, where the ranges of the corresponding interval eigenvalues are calculated by using the matrix perturbation theory. As pointed out in [13], the results in [12] are rather conservative. To reduce the conservatism, Lyapunov inequality is utilized for finding the maximum eigenvalue of a Hermitian matrix, and a new analytical robust stability checking method of FO-LTI uncertain system was provided in [13]. It should be noted that the obtained results in [12,13] are sufficient conditions. By using the complex Lyapunov inequality, a necessary and sufficient condition for the robust stability of FO-LTI interval systems with order $1 \leqslant \alpha < 2$ was established in [14]. The sufficient and necessary stability conditions of FO-LTI interval systems with order $\alpha : 0 < \alpha < 1$ and $1 \leqslant \alpha < 2$ in [15,16] were proposed in terms of linear matrix inequality (LMI), which were very convenient to be utilized in practice. Since FO-LTI interval systems investigated by the above-mentioned results [12–16] cannot exactly describe the FO-LTI systems with structured perturbations, paper [17] has presented necessary and sufficient conditions for the robust asymptotical stability of fractional-order linear systems with order $1 \leqslant \alpha < 2$ and structured perturbations. Robust stability for uncertain FO-LTI systems of order $\alpha : 0 < \alpha < 1$ with polytope-type uncertainty and the norm-bounded uncertainty were discussed in paper [18]. By using singular value decomposition and LMI techniques, robust control of fractional-order uncertain linear systems were considered in paper [19,20].

However, the above considered FO-LTI interval systems allow only independent or linearly dependent uncertain parameters. In fact, for geometric and inertia parameters in some physical dynamic system, the uncertain parameters usually exist in nonlinear form. Thus, the models based on the above uncertainty descriptions may seriously lead to "over bounds" of uncertainties, which may lead to robustness analysis results be too conservative [21]. In order to overcome the above mentioned conservation, a new uncertain state-space model which allows second-order uncertain parameters has been proposed in [21]. Then, such model has been extended to fractional-order case, and sufficient conditions in terms of LMIs for the stability and stabilization of such fractional-order model with $1 \leqslant \alpha < 2$ were presented in [22], where uncertainty appears in the form of a combination of additive uncertainty and multiplicative uncertainty. To the best of our knowledge, there are few results about robust stability and stabilization for fractional-order uncertain systems with second-order nonlinear uncertain parameters which are formulated in terms of a polytope of matrices. With the above motivation, robust stability and stabilization for such fractional-order polytopic type uncertain systems with order $\alpha : 0 < \alpha < 1$ and $1 \leqslant \alpha < 2$ are investigated respectively in the paper, some LMIs criteria for the robust stability and stabilization of such system are proposed respectively. Finally, two illustrative examples are given to show the effectiveness of the proposed results.

The remainder of this paper is organized as follows. In Section 2, some necessary definitions, lemmas are presented. Main results are proposed in Section 3. In Section 4, two simple numerical example are used to illustrate the validity and feasibility of the proposed method. Finally, conclusions are drawn in Section 5.

Notations

Throughout this paper, R^n and $R^{n\times m}$, denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices; M^T denotes transpose of matrix M; The notation $M > 0 (M < 0)$ means that the matrix M is positive (negative) definite; $\otimes$ is the Kronecker product of two matrices and $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$; $Sym\{M\}$ is used to denote the expression $M + M^T$, and $\star$ is used to denote a block matrix element that is induced by transposition. $\|\cdot\|$ represents any matrix norm.

2. Model description and preliminaries

In this paper, the well-known Caputo definition for fractional derivative is adopted, which allows utilization of initial values of classical integer-order derivatives with known physical interpretations. The Caputo (C) fractional derivative is defined as follows:

$$D^\alpha x(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} x^{(n)}(\tau) d\tau, \quad n-1 < \alpha < n.$$

Consider the following uncertain fractional-order mathematical model, which is the generalization of the integer-order one [21].

$$(A_n + \Delta_n) D^{n\alpha} y + (A_{n-1} + \Delta_{n-1}) D^{(n-1)\alpha} y + \cdots + (A_1 + \Delta_1) D^\alpha y + (A_0 + \Delta_0) y = F, \tag{1}$$

where α is order and belongs to $\alpha : 0 < \alpha < 1$ or $1 \leqslant \alpha < 2$. $A_i \in R^{m\times m}$ are known matrices which represent the values of the system at nominal working point. $\Delta_i \in R^{m\times m}$ is the unknown matrix representing the uncertain parameters. Vector F represents a known driving source. $D^{i\alpha} y (i = 1, 2, \ldots, n)$ denotes the $i\alpha$th order differential of vector y. A_n is assumed to be nonsingular and Δ_n satisfies $\|A_n^{-1}\Delta_n\| < 1$, which guarantee that $A_n + \Delta_n$ is nonsingular [23].

Denote $z_1(t) = y(t), z_2(t) = D^\alpha y(t), \ldots, z_n(t) = D^{(n-1)\alpha} y(t)$, it yields

$$\begin{cases} D^\alpha z_1(t) = z_2(t), \\ D^\alpha z_2(t) = z_3(t), \\ \qquad \vdots \\ D^\alpha z_{n-1}(t) = z_n(t), \\ D^\alpha z_n(t) = (A_n + \Delta_n)^{-1} [F - (A_0 + \Delta_0) z_1(t) \cdots - (A_{n-1} + \Delta_{n-1}) z_n(t)]. \end{cases} \tag{2}$$