Applied Mathematics and Computation 257 (2015) 374–380

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Pressure responses of a vertically hydraulic fractured well in a reservoir with fractal structure

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ARTICLE INFO

Keywords:
Vertically hydraulic fractured well
Fractal geometry
Fractal reservoir
Fractional derivatives

ABSTRACT

We obtain an analytical solution for the pressure-transient behavior of a vertically hydraulic fractured well in a heterogeneous reservoir. The heterogeneity of the reservoir is modeled by using the concept of fractal geometry. Such reservoirs are called fractal reservoirs. According to the theory of fractional calculus, a temporal fractional derivative is applied to incorporate the memory properties of the fractal reservoir. The effect of different parameters on the computed wellbore pressure is fully investigated by various synthetic examples.

1. Introduction

Hydraulic fracturing plays an important role in improving the productivity of damaged wells and wells producing. The vertical plane fracture is created by injecting fluid into the formation and then filling with proponing agents, such as propants, to prevent closure. In practical terms, two types of fractured well are considered: infinite (high) or finite (low) conductivity vertical fracture. In case of infinite conductivity fracture, it is assumed that the fluid flows along the fracture without any pressure drop. Finite conductivity fracture occurs when the pressure drop along the fracture plane is not negligible.

The classical diffusion equation has been used to explain the pressure responses of a well in a reservoir, which is assumed to be homogenous at all scales. However, recent studies show that the homogeneity assumption is not valid in most cases [1–7]. Due to this fact, fractal geometry has been used as an effective tool to describe the heterogeneities of these reservoirs, which are called fractal reservoirs [8–11]. Since the diffusion process of fractal reservoirs is history dependent, and the anomalous diffusion properties of fractal reservoirs cannot be fully described by the fractal model, the concept of fractional derivative has been used to incorporate the memory of the fluid flow [12–15]. Our main aim here is to analyze the pressure behavior of a well with an infinite conductivity vertical fracture in a fractal reservoir. An infinite radial system is considered in order to analyze the effects of different parameters on the well response.

The paper is organized as follows. After this brief introduction, the mathematical model is formulated in Section 2. A summary of the nomenclature used appears in Appendix A. The analytical solution to the model is provided in Section 3.

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http://dx.doi.org/10.1016/j.amc.2014.12.124
0096-3003/

Section 4 discusses how the well responses to different reservoir parameters. The main conclusions of our study are given in Section 5.

2. Model description

A schematic diagram of a vertically hydraulic fractured well is shown in Fig. 1. Fig. 2 shows the geometry of flow lines near the fractured well. Before discussing the mathematical model of transport process, we define three variables:

- the dimensionless pressure

$$p_D = \frac{2\pi k_w h(p_i - p(r,t))}{q\mu};$$

- the dimensionless time

$$t_D = \frac{k_w r_{wD}^{\theta} t}{\phi_w \mu c (2L_f)^2};$$

- the dimensionless radius

$$r_D = \frac{r}{2L_f}.$$

See Appendix A for the description of all the quantities involved. The best known and most useful model to describe the pressure behavior of fractal reservoirs was firstly proposed by Metzler et al. [12]. From here on we call *generalized diffusion equation* to the fractal-fractional diffusion (FFD) equation

$$\frac{1}{r_D^{\theta}}\frac{\partial^2 p_D}{\partial r_D^2} + \frac{\beta}{r_D^{\theta+1}}\frac{\partial p_D}{\partial r_D} = \frac{\partial^{\gamma} p_D}{\partial t_D^{\gamma}}, \tag{1}$$

where $\beta = d_{mf} - \theta - 1$. The parameter d_{mf} denotes the mass fractal dimension, while θ represents the conductivity index. Mass fractal dimension is responsible for the reservoir structure, and the conductivity index explains the diffusion process in the reservoir. The Caputo fractional order derivative is used to introduce $\partial^{\gamma} p_D / \partial t_D^{\gamma}$:

$$\frac{\partial^{\gamma} p_D}{\partial t_D^{\gamma}} = \frac{1}{\Gamma(m-\gamma)}\int_0^{t_D} (t_D - \tau)^{m-\gamma-1} p_D^{(m)}(\tau)d\tau, \tag{2}$$

where $\gamma \in \mathbb{R}^+$, $\lceil \gamma \rceil = m \in \mathbb{Z}^+$, and Γ represents the Gamma function, that is,

$$\Gamma(v) = \int_0^{\infty} e^{-\tau}\tau^{v-1}d\tau. \tag{3}$$

The order γ of the fractional derivative is related to the conductivity index by $\gamma = 2/(2+\theta)$.

3. Analytical solution

It is assumed that the pressure distribution of the reservoir is uniform and constant at initial time:

$$p_D(r_D, 0) = 0. \tag{4}$$

To obtain the line source solution of Eq. (1), we take $r_w \to 0^+$. The inner boundary condition without wellbore storage and skin effects can then be written as

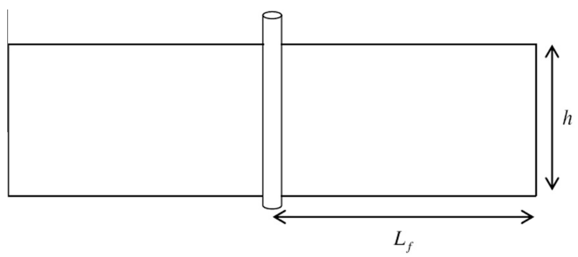

Fig. 1. Geometry of a vertically hydraulic fractured well.