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# Nonlinear dynamic response of a fractionally damped cylindrical shell with a three-to-one internal resonance

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ABSTRACT

Nonlinear vibrations of a cylindrical shell embedded in a fractional derivative viscoelastic medium and subjected to the different conditions of the internal resonance are investigated. The viscous properties of the surrounding medium are described by Riemann–Liouville fractional derivative. The displacement functions are determined in terms of eigenfunctions of linear vibrations of a free-simply supported shell. A novel procedure resulting in decoupling linear parts of equations is proposed with the further utilization of the method of multiple scales for solving nonlinear governing equations of motion by expanding the amplitude functions into power series in terms of the small parameter and different time scales. It is shown that the phenomenon of internal resonance can be very critical, since in a circular cylindrical shell the one-to-one, two-to-one, and three-to-one internal resonances, as well as additive and difference combinational resonances are always present. The three-to-one internal resonance is analyzed in detail. Since the internal resonances belong to the resonances of the constructive type, i.e., all of them depend on the geometrical dimensions of the shell under consideration and its mechanical characteristics, that is why such resonances could not be ignored and eliminated for a particularly designed shell. It is shown that the energy exchange could occur between two or three subsystems at a time: normal vibrations of the shell, its torsional vibrations and shear vibrations along the shell axis. Such an energy exchange, if it takes place for a rather long time, could result in crack formation in the shell, and finally to its failure. The energy exchange is illustrated pictorially by the phase portraits, wherein the phase trajectories of the phase fluid motion are depicted.



## 1. Introduction

The interest of researchers to the problems of the internal resonance in mechanical systems was initiated by the paper by Witt and Gorelik [1], wherein the authors showed theoretically and experimentally the two-to-one internal resonance with the energy exchange from one subsystem to another using the simplest two-degree-of-freedom mechanical system, and such interest does not relax up till now. It will suffice to mention the state-of-the-art article [2] and the monograph [3] involving the extensive review of literature in the field of internal resonances in different mechanical systems. Different

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types of the internal resonance: one-to-one, two-to-one, three-to-one, as well as a variety of combinational resonances, when three and more natural modes interact, have been discussed. The enumerated internal resonances were investigated in mechanical systems with more than one degree-of-freedom, as well as in strings, beams, plates, and shells.

It has been emphasized by many researchers [4–6] that the phenomenon of internal resonances can be very critical especially for circular cylindrical shells. Thus, the nonlinear vibrations of infinitely long circular cylindrical shells under the conditions of the two-to-one internal resonance were studied in [7] via the method of multiple time scales using the simple plane strain theory of shells. Parametrically excited vibrations of infinitely long cylindrical shells and nonlinear forced vibrations of a simply supported, circular cylindrical shell filled with an incompressible, inviscid, quiescent and dense fluid were investigated, respectively, in [4–6] using Donnell's nonlinear shallow-shell theory. The flexural deformation is usually expanded by using the linear shell eigenmodes, in so doing the flexural response involves several nodal diameters and one or two longitudinal half-waves. Internal resonances of different types have been analyzed in [8,9].

The extensive review of studies on shallow shells nonlinear vibrations could be found in the state-of-the-art articles by Kubenko and Koval'chuk [10], Amabili and Païdoussis [11], and Lee [12], as well as in a recent paper [13]. In spite of the fact that many studies have been carried out on large amplitude vibrations of circular cylindrical shells and many different approaches to the problem have been used, we agree with Amabili et al. [6] that this research area is still far from being well understood.

The problem of free as well as forced nonlinear vibrations of cylindrical shells can be considered from different positions depending on the shell geometry, in so doing the nonlinear displacement field is approximated by a finite sum of global interpolating functions. However the choice of appropriate modal expansions is fundamental for guaranteeing an accuracy of the results for large-amplitude vibrations [14]. Thus, for example, different expansions involving from 14 to 48 generalized co-ordinates, associated to natural modes of simply supported shells, have been discussed in [15].

A comparison of five shell theories for large-amplitude vibrations of circular cylindrical shells which are generally applied to geometrically nonlinear problems that use only three variables, which are the three middle surface displacements, has been carried out in [15]. More complicated shell theories suitable for moderately thick laminated shells exist, and they use five independent variables, three displacements and two rotations, [16,17] or even six variables if thickness variation is taken into account [18]. However, it has been emphasized in [15] that the majority of researchers utilize the Donnell–Mushtari–Vlasov type equations. Moreover, in the above mentioned papers, the internal resonance has been found within some combination of natural frequencies (1:1, 1:2, and 1:1:2) of one and the same type of vibrations, namely: flexural vibrations, regardless of the number of equations used for the analysis of the shell dynamic response [17,19].

Another type of the internal resonance was investigated in [20,21] during study of dynamic response of a nonlinear rectangular plate, when one frequency of the in-plane vibrations was equal (a 1:1 internal resonance) or twice as larger (a 1:2 internal resonance) than some frequency of the out-of-plane vibrations, i.e., the internal resonance between two different subsystems. The combinational resonances of the additive and difference type between two different subsystems were also investigated in [21].

In recent years much attention is given to damping features of mechanical systems subjected to the conditions of different internal resonances. Damping properties of nonlinear systems are described mainly by the first-order time-derivative of a generalized displacement [3]. However, as it has been shown by Rossikhin and Shitikova [22], who analyzed free damped vibrations of suspension combined system under the conditions of the one-to-one internal resonance, for good fit of the theoretical investigations with the experimental results it is better to describe the damping features of nonlinear mechanical systems in terms of fractional time-derivatives of the generalized displacements [23]. The analysis of nonlinear vibrations of a two-degree-of-freedom mechanical system, the damping features of which are described by a fractional derivative, has shown [24] that in the case when the system is under the conditions of the two-to-one or one-to-one internal resonance, viscosity may have a twofold effect on the system: a destabilizing influence producing unsteady energy exchange, and a stabilizing influence resulting in damping of the energy exchange mechanism. The same phenomenon was observed when considering nonlinear vibrations of a fractionally damped plate under the conditions of the two-to-one [20] or one-to-one [21] internal resonance.

It should be emphasized that a rich variety of papers dealing with fractional calculus, which has a long history [25,26], and its application in different fields of science and engineering have appeared in scientific literature during recent decades, among them reviews [23,27–33] and monographs [34–40]. As this takes place, the object of ever-increasing interest in fractional calculus application is fractional calculus viscoelasticity [23], and the present paper is a new contribution in the field.

In the given paper, nonlinear dynamic response of a thin cylindrical shell vibrating in a fractionally damped medium is investigated. The dynamic behavior of the shell is described by a set of three coupled nonlinear differential equations with due account for the fact that the shell is being under the conditions of the internal resonance resulting in the interaction of modes corresponding to the mutually orthogonal displacements. The displacement functions are determined in terms of eigenfunctions of linear vibrations. A new procedure resulting in decoupling linear parts of equations is proposed with the further utilization of the method of multiple scales for solving nonlinear governing equations of motion, in so doing the amplitude functions are expanded into power series in terms of the small parameter and depend on different time scales. It is shown that the phenomenon of the internal resonance between vibrational subsystems of the cylindrical shell under consideration can be very critical, since in the circular cylindrical shell of such a type the two-to-one, one-to-one, three-to-one internal resonance, as well as additive and difference combinational resonances are always present, in so doing