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On differences and similarities in the analysis of Lorenz, Chen, and Lu systems

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ABSTRACT

Currently it is being actively discussed the question of the equivalence of various Lorenz-like systems and the possibility of universal consideration of their behavior (Algaba et al., 2013a,b, 2014b,c; Chen, 2013; Chen and Yang, 2013; Leonov, 2013a), in view of the possibility of reduction of such systems to the same form with the help of various transformations. In the present paper the differences and similarities in the analysis of the Lorenz, the Chen and the Lu systems are discussed. It is shown that the Chen and the Lu systems stimulate the development of new methods for the analysis of chaotic systems. Open problems are discussed.

1. Introduction

Currently it is being actively discussed the question of the equivalence of various Lorenz-like systems and the possibility of universal consideration of their behavior [2–5,11,14,35] in view of the possibility of reduction of such systems to the same form with the help of various transformations.

In the present paper the differences and similarities in the analysis of these systems are discussed and it is shown that the Chen and the Lu systems stimulate for the development of new methods for the analysis of chaotic systems.

2. Lorenz-like systems: Lorenz, Chen, Lu, and Tigan systems

Consider the famous Lorenz system [60]

$$\begin{aligned} \dot{x} &= \sigma(y - x), \\ \dot{y} &= \rho x - y - xz, \\ \dot{z} &= -\beta z + xy, \end{aligned} \tag{1}$$

where $\sigma,\ \rho,\ \beta$ are positive parameters.

Consider the Chen system [12]

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$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= (c - a)x + cy - xz, \\ \dot{z} &= -bz + xy \end{aligned} \tag{2}$$

and the Lu system [61]

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= cy - xz, \\ \dot{z} &= -bz + xy, \end{aligned} \tag{3}$$

where a, b, c are real parameters. Systems (2) and (3) are Lorenz-like systems, which have been intensively studied in recent years.

In 2012 G.A. Leonov suggested to consider the following substitutions [35]

$$x \to hx, \quad y \to hy, \quad z \to hz, \quad t \to h^{-1}t \tag{4}$$

with $h = a$. By this transformation for $a \neq 0$ one has in (2) and (3)

$$a \to 1, \quad c \to \frac{c}{a}, \quad b \to \frac{b}{a}.$$

For $a = 0$ the Chen and the Lu systems become linear and their dynamics have minor interest. Thus, without loss of generality, one can assume that $a = 1$. Remark that the transformation (4) with $h = a$ does not change the direction of time for the positive chaotic parameters considered in the works [12,61].

Later, in 2013, the transformation (4) was independently considered in the works [2,3][1] with $h = -c$ for the reduction of the Chen system (2):

$$\begin{aligned} \dot{x} &= -\frac{a}{c}(y - x), \\ \dot{y} &= \left(\frac{a}{c} - 1\right)x - y - xz, \quad \sigma = -\frac{a}{c}, \quad \rho = \frac{a}{c} - 1, \quad \beta = -\frac{b}{c} \quad (\sigma + \rho = -1), \\ \dot{z} &= \frac{b}{c}z + xy, \end{aligned} \tag{5}$$

and the Lu system (3):

$$\begin{aligned} \dot{x} &= -\frac{a}{c}(y - x), \\ \dot{y} &= -y - xz, \quad \sigma = -\frac{a}{c}, \quad \rho = 0, \quad \beta = -\frac{b}{c} \quad (\rho = 0), \\ \dot{z} &= \frac{b}{c}z + xy, \end{aligned} \tag{6}$$

to the form of the Lorenz system (1).

Note that here in contrast to the previous transformation: 1) the transformation (4) with $h = -c$ change the direction of time for $c > 0$ considered in the works [12,61], 2) for $c = 0$ the Chen and the Lu systems do not become linear and their dynamics may be of interest. For $c < 0$ the transformation (4) with $h = -c$ does not change the direction of time.

For $c = 0$ in [2,3] it is suggested to apply the previous transformation (4) with $h = a$ and is claimed that the Chen and the Lu systems with $c = 0$ are "*a particular case of the T-system*" [26,77] (which was published later)

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= (c - a)x - axz, \\ \dot{z} &= -bz + xy. \end{aligned} \tag{7}$$

To fill the formal gap in the notation of the parameters in this case it is required the additional transformation $x \to x/\sqrt{a}$, $y \to y/\sqrt{a}$, $z \to z/a$. Finally, to transform the Chen and the Lu systems with $c = 0$ to the T-system one has to apply the following transformation

$$x \to \sqrt{a}x, \quad y \to \sqrt{a}y, \quad z \to z, \quad t \to a^{-1}t. \tag{8}$$

For $\sigma = 10$, $\beta = 8/3$ and $0 < \rho < 1$, the Lorenz system is stable. For $1 < r < 24.74\cdots$ the zero fixed point looses its stability and two additional stable fixed points appear. For $\rho > 24.74\cdots$ all three fixed points become unstable and trajectories, depending on the initial data, may be repelled by them in a very complex way. For the parameter set

[1] Submission dates: [35] – December 27, 2012; [2] – 22 January 2013; [3] – 22 January 2013.