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# Fractional stochastic Volterra equation perturbed by fractional Brownian motion

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ABSTRACT

In this paper, we consider a class of fractional stochastic Volterra equation of convolution type driven by infinite dimensional fractional Brownian motion with Hurst index $h \in (0, 1)$. Base on the explicit formula for the scalar resolvent function and the properties of the Mittag–Leffler's function, the existence and regularity results of the stochastic convolution process are established. Separate proofs are required for the cases of Hurst parameter above and below $\frac{1}{2}$ and it will turn out that the regularity of the solution increases with Hurst parameter $h$.



## 1. Introduction

The subject of stochastic calculus with respect to fractional Brownian motion (fBm) has gained considerable popularity and importance due to its frequent appearance in a wide variety of physical phenomena, such as hydrology, economic, telecommunications and medicine. Many contributions for stochastic calculus with respect to fBm have been emerged in the last decades, see [3,10,22,25,26].

Since some physical phenomena are naturally modeled by stochastic partial differential equations or stochastic integro-differential equations and the randomness can be described by a fBm, it is important to study the problems of solutions of infinite dimensional equations with a fBm. Many studies of the solutions of stochastic equations in an infinite dimensional space with a fBm have been emerged recently, (see [1–4,7,8,10–21,23,24,27,29,30,32–36]).

Stochastic integro-differential equations with respect to fractional kernel have been extensively researched in recent years, as they make a good link between the heat equation and the wave equation [5,6,9,28].

P. Clément, G. Da Prato and J. Prüss [9] considered white noise perturbation of the equation of linear parabolic viscoelasticity. The authors shown that, under suitable assumptions, the stochastic convolution leads to regular solutions and the samples are Hölder continuous. S. Bonaccorsi and L. Tubaro [5] studied a class of stochastic linear Volterra equations of convolution type defined by fractional integration kernels $\frac{t^{\rho-1}}{\Gamma(\rho)}, \rho \in (0, 2)$. By using the explicit formula for the scalar resolvent function, the authors established the basic properties of the stochastic convolution process. Stefan Sperlich and Mathias Wilke [31] investigated fractional white noise perturbation of parabolic Volterra equations. They extended the results of [9] on the fractional white noise perturbation and obtained the regularity of the solution.

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The purpose of this paper is to consider a class of Volterra equation driven by infinite dimensional fractional white noise with Hurst parameter $h \in (0,1)$. Let $H$ denotes a separable Hilbert space with norm $\|\cdot\|$ and inner product $(\cdot,\cdot)$. Let $A$ be a closed linear densely defined operator in $H$. Consider the stochastic Volterra equation

$$\begin{cases} u(t) = g_\rho * Au(t) + g_\theta * \dot{B}^h(t), & t \in [0,1], \\ u(t) = u_0 \in H, \end{cases} \tag{1.1}$$

where $(a* \ b)(t) = \int_0^t a(s)b(t-s)\mathrm{d}s$ denotes the convolution of $a$ and $b$, $g_\rho(t) = \frac{t^{\rho-1}}{\Gamma(\rho)}$ denotes the fractional integration kernel for any $\rho > 0$ and $B^h$ is a fractional Brownian motion in $H$ with Hurst parameter $h \in (0,1)$ with corresponding fractional white noise $\dot{B}^h$ (see Section 2 below).

When $h = \frac{1}{2}$, problem (1.1) becomes a linear Volterra equation perturbed by white noise, which has been studied by S. Bonaccorsi and L. Tubaro [5]. Thus the work of [5] is a special case of our achievements. A special cases of problem (1.1) with $\theta = 1$ has been studied by Bonaccorsi in a recent paper [6], in which the author obtained the existence of a mild solution. Problem (1.1) was also mentioned in [31] in regard to existence and regularity of a stochastic convolution. Since problem (1.1) related to explicit solutions via Mittag–Leffler's function and the properties of Mittag–Leffler's function are used, we obtain more precise estimates on the stochastic convolution.

The aim of this paper is to obtain the existence, uniqueness and regularities of the mild solution of problem (1.1). Our proofs are base on the explicit formula for the scalar resolvent function and the properties of the Mittag–Leffler's function. Separate proofs are required for the cases of Hurst parameter above and below $\frac{1}{2}$ and it will turn out that the regularity of the solution increases with Hurst parameter $h$.

The choice of the kernel $g_\rho$ in problem (1.1) is motivated by the fact that applying formally the fractional derivative $D^\rho$ to (1.1) with $\rho = \theta \in (0,2)$, we obtain a fractional stochastic differential equation

$$D^\rho u(t) = Au(t) + \dot{B}^h(t). \tag{1.2}$$

In the first part of this paper, we deal with the case of $\theta = 1$ in (1.1), that is

$$u(t) = g_\rho * Au(t) + B^h(t). \tag{1.3}$$

Then we will extend the results to any $1 - h < \theta < 2$.

Throughout the paper, we use the letter $C$ denotes a constant that may not be the same form one occurrence to anther, even in the same line. We express the dependence on some parameters by writing the parameters as arguments, e.g. $C = C(h)$.

The remaining of this paper is organized as follows. In Section 2, we give some assumptions of stochastic Volterra Eq. (1.3) and preliminaries of the Wiener integral with fBm. In Section 3, we prove the existence of the stochastic convolution. In Section 4, we consider the Hölder continuity of the stochastic convolution of (1.3). Finally in Section 5, we give the existence and regularity results of (1.1).

## 2. Preliminaries

In this section, a fBm in a separable Hilbert space is introduced, a Wiener-type integral with respect to this process is defined. We also recall some basic results concerning Volterra equations perturbed by fractional white noise.

**Assumption 2.1.** A is a self-adjoint, negative defined operator on $H$; there exists a basis $\{e_k, k \in \mathbb{N}\}$ of $H$ such that

$$Ae_k = -\mu_k e_k, \quad k \in \mathbb{N}, \tag{2.1}$$

where $\{\mu_k\}$ is an increasing sequence of positive real numbers and $\lim_{k\to\infty}\mu_k = +\infty$.

As shown in [5], if the conditions of Assumption 2.1 are valid, (1.3) admits a resolvent operator $S(t), t \geqslant 0$ defined by

$$S(t)x = x + (g_\rho * AS(\cdot)x)(t), \quad x \in D(A), t \geqslant 0. \tag{2.2}$$

By means of the spectral decomposition of $A$, the resolvent operator $S(t)$ can be defined by

$$S(t)e_k = s_{\mu_k} e_k, \quad k \in \mathbb{N}, \tag{2.3}$$

where $s_\alpha(t)$ is the solution of the scalar resolvent equation

$$s_\alpha(t) + \alpha \int_0^t g_\rho(t-\tau)s_\alpha(\tau)\mathrm{d}\tau = 1, \quad t \geqslant 0, \alpha \geqslant 0. \tag{2.4}$$

Let us cite some useful properties of $s_\alpha$ which were proven in [5]. Recall that the Mittag–Leffler's function $E_\rho(z)$ is defined by the series

$$E_\rho(z) = \sum_{k=0}^{\infty} \frac{(-1)^k z^k}{\Gamma(\rho k + 1)}, \quad \rho > 0, z \in \mathbb{R}. \tag{2.5}$$