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Dynamics of stochastic delay Lotka–Volterra systems with impulsive toxicant input and Lévy noise in polluted environments

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ABSTRACT

In this paper, two stochastic delay Lotka–Volterra systems (i.e., competition system and predator–prey system) with impulsive toxicant input and Lévy noise in polluted environments are proposed and investigated. Under some simple assumptions, sufficient and necessary criteria for stability in time average and extinction of each population are established. The thresholds between stability in time average and extinction of each model are obtained. Some recent results are improved and extended greatly.

1. Introduction

Recently, stochastic Lotka–Volterra model has been widely investigated (see e.g. [1–16] and the references cited therein). A stochastic two-species Lotka–Volterra model with time delays takes the following form:

$$\begin{cases} dx_1(t) = x_1(t)[r_1 - a_{11}x_1(t) - a_{12}x_2(t-\tau_1)]dt + \sigma_1 x_1(t)dB_1(t), \\ dx_2(t) = x_2(t)[r_2 - a_{21}x_1(t-\tau_2) - a_{22}x_2(t)]dt + \sigma_2 x_2(t)dB_2(t), \end{cases} \tag{1}$$

with initial conditions

$$x_i(s) = \varphi_i(s) > 0, \quad s \in [-\tau, 0]; \quad \varphi_i(0) > 0, \quad i = 1, 2, \tag{2}$$

where $x_i(t)$ represents the population size of the ith species at time $t; r_i, a_{ij}$ and σ_i $(i, j = 1, 2)$ are constants; $\tau_i \geqslant 0, \tau = \max\{\tau_1, \tau_2\}$ and $\varphi_i(s)$ $(i = 1, 2)$ is a continuous function on $[-\tau, 0]$; $B(t) = (B_1(t), B_2(t))^T$ denotes a standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t \in R_+}$ satisfying the usual conditions. Liu and Wang [3] investigated model (1) in the competition case (that is $r_1 > 0$, $r_2 > 0, a_{12} > 0$ and $a_{21} > 0$). They discussed the stability in time average and the extinction of the system. Furthermore, Liu et al. [4] studied model (1) in the predator–prey case (that is $r_1 > 0, r_2 < 0, a_{12} > 0$ and $a_{21} < 0$). The authors considered the stability in the mean and extinction of the model.

As far as we known, environmental pollution has been one of the most important social problems today. Due to toxins in the environment, many species have been extinctive and some of them are on the verge of extinction. So controlling the environmental pollution and the conservation of biodiversity have been the major topics of all the countries. This inspires researchers to investigate the influences of toxins on populations.

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In recent years, many population systems in polluted environments have been proposed and discussed (see e.g. [17–37] and the references cited therein). For example, Liu [28] studied the following two-species Lotka–Volterra predator–prey system with impulsive toxicant input in polluted environments:

$$\begin{cases} dx_1(t) = x_1(t)[r_{10} - r_{11}C_{10}(t) - a_{11}x_1(t) - a_{12}x_2(t-\tau_1)]dt + \alpha_1 x_1(t)dB_1(t), \\ dx_2(t) = x_2(t)[-r_{20} - r_{21}C_{20}(t) - a_{22}x_2(t) + a_{21}x_1(t-\tau_2)]dt + \alpha_2 x_2(t)dB_2(t), \\ \frac{dC_{10}(t)}{dt} = k_1C_e(t) - (g_1+m_1)C_{10}(t), \\ \frac{dC_{20}(t)}{dt} = k_2C_e(t) - (g_2+m_2)C_{20}(t), \\ \frac{dC_e(t)}{dt} = -hC_e(t), \\ t \neq n\gamma, \quad n \in Z^+, \\ \Delta x_i(t) = 0, \quad \Delta C_{i0}(t) = 0, \quad \Delta C_e(t) = b, \\ t = n\gamma, \quad n \in Z^+, \quad i = 1,2, \end{cases} \tag{3}$$

with initial condition

$$x_i(t) = \phi_i(t) > 0, \quad t \in [-\tau, 0]; \quad \phi_i(0) > 0, \quad i = 1,2, \tag{4}$$

where all the parameters are positive constants, $\tau_i \geqslant 0$, $\tau = \max\{\tau_1, \tau_2\}, \phi_i(t)$ is continuous on $[-\tau, 0], \Delta f(t) = f(t^+) - f(t), Z^+ = \{1,2,\ldots\}$, $x_1(t)$ and $x_2(t)$ represents the density of prey population and the predator population at time t, respectively, r_{i0} denotes the intrinsic growth rate of the ith population without toxicant, r_{i1} represents the ith population response to the pollutant present in the organism, $C_{i0}(t)$ denotes the concentration of toxicant in the ith organism, $C_e(t)$ denotes the concentration of toxicant in the environment, $kC_e(t)$ denotes the organism's net uptake of toxicant from the environment, $gC_{i0}(t) + mC_{i0}(t)$ represents the egestion and depuration rates of the toxicant in the ith organism, $hC_e(t)$ denotes the toxicant loss from the environment itself by volatilization and so on, γ stands for the period of the impulsive influence about the exogenous input of toxicant, b denotes the toxicant input amount at every time, $\dot{B}_i(t)$ is a white noise and α_i^2 is the intensity of the noise.

On the other hand, in the real world, population systems may subject to sudden and severe environmental perturbations, such as earthquakes, epidemics, harvesting and so on. These phenomena can not be described better by system (1). Introducing Lévy noises into the corresponding population systems may be a good way to describe these phenomena (see e.g. [38–43]). In [38], Bao et al. initially proposed and studied stochastic competitive Lotka–Volterra population dynamics with jumps. Then Bao and Yuan [39] studied a general Lotka–Volterra population dynamics driven by Lévy noise. Under some conditions, they investigated some important properties of the system. These important results show that jump processes have significant effects on the properties of systems. Recently, Liu and Wang [40] have investigated the following stochastic Lotka–Volterra model of two interacting species with jumps:

$$\begin{cases} dx_1(t) = x_1(t^-)\left\{[r_1 - a_{11}x_1(t^-) - a_{12}x_2(t^-)]dt + \sigma_1 dB(t) + \int_{\mathbb{Y}} \gamma_1(u)\widetilde{N}(dt,du)\right\}, \\ dx_2(t) = x_2(t^-)\left\{[r_2 - a_{21}x_1(t^-) - a_{22}x_2(t^-)]dt + \sigma_2 dB(t) + \int_{\mathbb{Y}} \gamma_2(u)\widetilde{N}(dt,du)\right\}, \end{cases}$$

where $x_i(t^-)$ denotes the left limit of $x_i(t), i = 1,2$, N represents a Poisson counting measure with characteristic measure λ on a measurable subset $\mathbb{Y}$ of $(0,\infty)$ with $\lambda(\mathbb{Y}) < \infty$ and $\widetilde{N}(dt,du) = N(dt,du) - \lambda(du)dt$. The authors [40] studied some important asymptotic properties of the system and obtained some interesting results. However, as we all know, time delays frequently occur in almost every situation and all species should exhibit time delay, Kuang [44] has revealed that ignoring time delays means ignoring reality. Hence it is essential to take time delays into account. Strongly inspired by the above mentioned works, in this paper, we consider the following stochastic delay Lotka–Volterra systems with impulsive toxicant input and Lévy noise in polluted environments:

$$\begin{cases} dx_1(t) = x_1(t^-)\left\{[r_{10} - r_{11}C_{10}(t) - a_{11}x_1(t^-) - a_{12}x_2(t^- - \tau_1)]dt + \sigma_1 dB_1(t) + \int_{\mathbb{Y}} \gamma_1(u)\widetilde{N}(dt,du)\right\}, \\ dx_2(t) = x_2(t^-)\left\{[r_{20} - r_{21}C_{20}(t) - a_{21}x_1(t^- - \tau_2) - a_{22}x_2(t^-)]dt + \sigma_2 dB_2(t) + \int_{\mathbb{Y}} \gamma_2(u)\widetilde{N}(dt,du)\right\}, \\ \frac{dC_{10}(t)}{dt} = k_1C_e(t) - (g_1+m_1)C_{10}(t), \\ \frac{dC_{20}(t)}{dt} = k_2C_e(t) - (g_2+m_2)C_{20}(t), \\ \frac{dC_e(t)}{dt} = -hC_e(t), \\ t \neq n\gamma, \quad n \in Z^+, \\ \Delta x_i(t) = 0, \quad \Delta C_{i0}(t) = 0, \quad \Delta C_e(t) = b, \\ t = n\gamma, \quad n \in Z^+, \quad i = 1,2, \end{cases} \tag{5}$$

with initial conditions

$$x_i(s) = \varphi_i(s) > 0, \quad s \in [-\tau, 0]; \quad \varphi_i(0) > 0, \quad i = 1,2, \tag{6}$$