Applied Mathematics and Computation 256 (2015) 75–82

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stability analysis of complex-valued impulsive systems with time delay

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ARTICLE INFO

Keywords:
Complex-valued impulsive systems
Time delay
Stability
Razumikhin technique
Lyapunov technique

ABSTRACT

In this paper, the global exponential stability of complex-valued impulsive systems is addressed. Some new sufficient conditions are obtained to guarantee the global exponential stability by the Lyapunov–Razumikhin theory, which extend and improve most of recent results. Moreover, the obtained Razumikhin conditions are very simple and efficient to verify in real problems and helpful to investigate the stability of delayed neural networks and synchronization problems of chaotic systems under impulsive perturbation. Finally, a numerical example is given to show the effectiveness of the obtained results.
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1. Introduction

Impulsive effects exist widely in the world. As we all have known, the state of systems are often subject to instantaneous disturbances and experience abrupt changes at certain instants, which might be caused by frequent changes or other suddenly noises. These systems are called impulsive systems, which are governed by impulsive differential equations or impulsive difference equations.

In the past decades, the study of impulsive system theory has attracted the interest of many researchers due to its widely applications in many fields of modern science such as computer science, communication, economy, biology, as well as artificial neural systems. So far, many profound results have been obtained for the dynamics of impulsive systems, especially the impulsive systems with mixed impulse moments [2–8]. However, almost all of the existing works only focus on the real-valued impulsive systems. In fact, many classical systems are subject to study in complex-valued field, such as quantum system [9–12], Ginzburg–Landau equation [16], Orr–Sommerfeld equation [17], complex Riccati equation [18] and complex Lorenz equation [19]. In [13–15] the authors have done some work about the complex-valued neural network, which have been extended the scope of applications in optoelectronics, filtering, imaging, speech synthesis, computer vision, remote sensing, quantum devices. Hence, it is significative and important to study on the properties of the complex-valued impulsive differential systems.

Recently,several interesting results concerning the dynamical properties of the complex-valued impulsive systems have been proposed [1,20–23,28–30]. In [1], the authors obtained some sufficient conditions for existence and uniqueness of solutions to complex-valued nonlinear impulsive systems. Ref. [28] studied the local asymptotically stability of complex-valued impulsive systems. [29] the issue of controllability and observability for a class of time-varying impulsive systems defined in

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http://dx.doi.org/10.1016/j.amc.2015.01.006
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complex fields. Fu [30] is devoted to a study of the null controllability for the semilinear parabolic equation with a complex principal part. But to the best of our knowledge, there has been few reports about the exponential stability of complex-valued impulsive systems. In practice, the exponential stability property is often expected. For example, to solve various optimization problems, it is desirable that there is only one equilibrium point which is exponentially stable so as to avoid the risk of having spurious equilibria and being trapped at local minima [20,21]. Also, the exponential stability property is particularly important when the exponential convergence rate is used to determine the speed of neural computations [22,23]. Thus, it is necessary and important to investigate exponential stability and to estimate the exponential convergence rate of complex-valued impulsive function differential equations from the view of both theory and application, however, there few study of the complex-valued impulsive system. The main contribution of this paper are as follows:

(1) A more general model both consider complex-valued and impulsive obtained.
(2) Some easy to check stability criteria are analyzed mathematically and by simulations.
(3) The asymptotical stability of complex-valued impulsive systems has been addressed in [28], and existence and uniqueness of solutions in [1]. Compared with those results, in this paper, the global exponential stability has been obtained with an exponential convergent rate.

Motivated by the above discussions, in this paper, we shall investigate the exponential stability of complex-valued impulsive systems with delays. By employing the improved Razumikhin technique and Lyapunov functions, some new conditions ensuring the global exponential stability will be obtained. This paper is organized as follows. In Section 2, the complex-valued impulsive systems with delay to be deal with are formulated and several results about the complex-valued continuous functions are presented. In Section 3, the global exponential stability criteria of complex-valued impulsive systems is established and a example presented. Finally, some conclusions are drawn in Section 4.

2. Notations and preliminaries

Let $\mathbb{C}$ denote the set of complex-valued numbers. $\mathbb{R}$ the set of real-valued numbers. $\mathbb{R}^+$ the set of nonnegative real-valued numbers. $\mathbb{Z}^+$ the set of positive integers and $\mathbb{C}^n$ the n-dimensional complex-valued space equipped with the Euclidean norm $\|\cdot\|$. Let $\Omega \in \mathbb{C}^n$ be a neighborhood of the origin. $W(z) \in C[\Omega, R], W(0) = 0$. $\mathcal{K} = \{a \in C(\mathbb{C}^+, \mathbb{C}^+)$, where $a(0) = 0$ and $a(t) > 0$ for $t > 0$ and a is strictly increasing in t}. For any $t \geqslant t_0 \geqslant 0 > \alpha \geqslant -\infty$. $I = [t+\alpha, +\infty)$. In the case when $\alpha = -\infty$ the interval $[t+\alpha, t]$ is understood to be replaced by $(-\infty, t]$. Let D be an open set in PC $([\alpha, 0], \mathbb{C}^n)$, where PC $([\alpha, 0], \mathbb{C}^n)$=$\{\varphi : [\alpha, 0] \to \mathbb{C}^n$ is continuous everywhere except at finite number of points t, at which $\varphi(t^+), \varphi(t^-)$ exist and $\varphi(t^+) = \varphi(t)\}$. Define $\mathrm{PCB}(t) = \{z_t \in D : z_t$ is bounded}. For $\varphi \in PCB(t)$, the norm of φ is defined by $\|\cdot\| = \sup_{\alpha \leqslant \theta \leq 0} |\varphi(\theta)|$.

Let a plant be a general complex-value system

$$z' = f(t, z(\cdot)) \tag{1}$$

where $z \in \mathbb{C}^n$ is the state variable, $f : I \times \Omega \to \mathbb{C}^n$ is complex-valued continuous functions.

Definition 1. For a given plant (1), the sequence $\tau_k, I_k(z(\tau_k))$ is called a impulsive control law of (1), if there exits a set of control instant τ_k, and control laws $I_k(z(\tau_k)) \in \mathbb{C}^n$ such that the solution of the following impulsive system described by

$$\begin{cases} z'(t) = f(z, t) & t \geqslant t_0, t \neq \tau_k \\ z(\tau_k^+) = I_k(z(\tau_k^-)) & t = \tau_k \end{cases} \tag{2}$$

where $I_k(t, z) \in C([t_0, \infty) \times \mathbb{C}^n, \mathbb{C}^n)$ are complex-valued continuous functions, $\triangle z(\tau_k) = z(\tau_k^+) - z(\tau_k^-), f \in C([\tau_{k-1}, \tau_k] \times D, \mathbb{C}^n)$. For each $t \geqslant t_0, z_t \in D$ is defined by $z_t(s) = z(t+s), s \in [\alpha, 0]. I_k(t_k, 0) = 0, k = \mathbb{Z}^+$. And the impulsive instants τ_k satisfy

$$0 \leqslant t_0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots \lim_{k \to +\infty} \tau_k = +\infty. \tag{3}$$

For any $\sigma \geqslant 0$ and $\phi \in D$, the initial condition for system (2) is given by

$$z_\sigma(s) = \phi, \quad \alpha \leqslant s \leqslant 0.$$

Definition 2. $W(z)$ is called a complex positive definite function on Ω, if $W(z) \geqslant 0$ for any $z \in \Omega$, and $W(z) = 0$ if and only if $z = 0$.

Definition 3 [27]. Let $V : I \times \Omega \mapsto \mathbb{R}^+$, then V is said to belong to $\mathcal{V}_0$ if

(1) V is locally Lipschitzian in z, and $V(t, z) = 0$ if and only if $z = 0$;
(2) V is continuous in $(\tau_{k-1}, \tau_k] \times \Omega$ and for each $z \in \Omega,\ k = 1, 2, \ldots$

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