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Group-separations based on the repeated prisoners' dilemma games

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ABSTRACT

We model group-separations in an n-player set. In the n-player set, every two players play an infinitely repeated two-player prisoners' dilemma game. Each player takes a mixed strategy to play the game and *trigger strategy* is used to punish the deviator. Let all players share a common discount factor δ. We find that with the variation of δ, the n-player set is *separated* into several subsets such that (1) for any two players in any two different subsets, their strategy profile is not a subgame perfect equilibrium and (2) each subset cannot be separated into several subsets that satisfy (1). Such subsets are called *groups* and the *separation* is called *group-separation*. We aim to specify the intervals (of δ) such that group-separations emerge. Particularly, we focus on the relationship between the interval and the form of each group-separation.

1. Introduction

Let us consider the following scene: there are a number of persons and each of them has his/her individual behavior. Every two persons interact pairwise with their behaviors. Each pairwise interaction is blocked if one person in the interaction suffers a loss. Then, do the persons taking relatively similar behaviors group together? This paper is motivated by such issue.

This paper proposes a model of group-separations in an n-player set N. In N, every two players play an *infinitely repeated two-player prisoners' dilemma game* (or IRPD for short) and the IRPD is with perfect monitoring and discounting. Each player in N has his/her individual mixed strategy and the individual mixed strategy can be interpreted as the individual behavior mentioned above. The repeated game strategy profile (or strategy profile for short) of *trigger strategy* is that two players always take the individual mixed strategy repeatedly if there is no deviation occurring. Otherwise, after a player deviates, his/her opponent triggers the punishment to punish the player. Let all players share a common discount factor $\delta \in [0, 1)$. Then, for every two players, in their IRPD, there exists a real number $\tilde{\delta} \in [0, 1)$ such that for all $\delta \in [\tilde{\delta}, 1)$, the strategy profile of the two players is a *subgame perfect equilibrium* (or SPE for short) [1]. This also implies that when $\delta \in [0, \tilde{\delta})$, the strategy profile of the two players is not an SPE. $\tilde{\delta}$ is the lower bound of the interval of the common discount factor δ supporting SPE (or the lower bound of SPE for short). Note that different pairs of players may have different lower bound of SPE. Thus, it can be inferred that for each $\delta \in [0, 1)$, the strategy profiles of some pairs of players in N is an SPE meanwhile the others are not.

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Following the above model, we find that with the variation of δ, the n-player set N is *separated* into several subsets such that (1) for any two players in any two different subsets, their strategy profile is not an SPE and (2) each subset cannot be separated into several subsets that satisfy (1) (see Section 3). Such subsets are called *groups* and the *separation* is called *group-separation*. We specify the intervals (of δ) such that group-separations emerge in the n-player set N and focus on the relationship between the interval and the form of each group-separation (see Section 4). Particularly, we have another finding that when δ lies in some specific subintervals of the intervals of group-separations, the strategy profile of every two players in each group is an SPE. Such group and group-separation are called *complete group* and *complete-group-separation* respectively (see Section 5).

Furthermore, let the n-player set N also represent a set of n vertices of a graph and let E_N be a set of the edges such that every two vertices in N have an edge. Here, let each edge in E_N refer to the IRPD between the corresponding two players. Then, the structure of all IRPDs based on n-player set N can be expressed in the form of an undirected complete graph $[N, E_N]$ and each group of group-separations can be denoted by a subgraph of $[N, E_N]$. See Example 1 in Section 3.

In essence, the group-separation in n-player set can be interpreted that the n-player set is separated into several subsets and each subset forms a group. [7] constructs a model of group formation and uses the group-trigger strategy to deter each member in the group from defecting. However, the prisoners' dilemma in [7] is an n-player version.

In our paper, the formation of groups can be described by the formation of the subgraphs in an undirected complete graph. The term *graph formation* is usually called *network formation*. There are a number of literatures exploring network formation game. In the early researches such as [8–11], the network represents the structure of the interactions based on the set of players and each edge represents an interaction between the corresponding two players. In these researches, each edge (i.e., link) is formed if the corresponding player can benefit from forming the edge. These researches mainly study the stability of the forms of networks, which is different from our paper.

The structure of the model in this paper can be expressed by an undirected complete graph. It can also be interpreted that in the graph, each player lies in a vertex and he/she plays an IRPD with each neighbor. Such framework can be seen in a number of researches regarding evolutionary game theory. The recent researches [12,13] model the formation of edges (i.e., links) by the approach of evolutionary game. And [14–25] study the effect from the structure of networks on the evolution of cooperation. Different from these researches, in our paper, each player has his/her mixed strategy. Our goal is to study that how each player's mixed strategy affects the organization of all the players. Simply speaking, when the difference of the mixed strategies of two players is too big, they cannot have an equilibrium interaction and the non-equilibrium interaction will be "blocked" by trigger strategy. Such "blocking the interaction" yields a group-separation in the set of players.

The rest of the paper is organized as follows. In Section 2, we illustrate the framework of IRPD with perfect monitoring and discounting. Section 3 introduces the structure of the model. Section 4 and Section 5 mainly give the theorems regarding group-separation and complete-group-separation respectively. The conclusion is given in Section 6. We give a number of numerical examples to illustrate the definitions and theorems.

2. Stage game and repeated game

Following [2], we give the framework of stage game and infinitely repeated game with perfect monitoring and discounting.

Stage game is a two-player prisoners' dilemma game. The payoff matrix is given as Table 1 (see [3]).

In the two-player prisoners' dilemma game, two players are faced with the choice to *cooperate* or to *defect* (C or D for short). They make choices simultaneously and cannot exchange the information about their choices with each other. After they made their choices, each player receives his/her payoff. If both cooperate, their payoff R is higher than the payoff P received if both defect. But if one player defects while the other cooperates, then the defector's payoff T is higher than R, while the cooperator's payoff S is smaller than P. It is furthermore assumed that $R > (T + S)/2$, so that joint cooperation is more profitable than alternating C and D. Here, C and D are called pure-strategy action in the game.

For each player $l = i, j$, the mixed-strategy actions (or actions for short) set is given as $B_l = \{(\rho, 1-\rho) : \rho \in [0,1]\}$. Thus, for an action $b_l \in B_l$, $b_l = (p_l, 1-p_l)$ where p_l refers to the probability assigned to the strategy C. The set of all action profiles is given as $B = B_i \times B_j$. For an action profile $b \in B$, the stage game payoff for player l is denoted by $u_l(b)$ and $u(b) = (u_i(b), u_j(b))$ is the payoff vector. The feasible payoffs set $\mathcal{F}$ of stage game is defined as $\mathcal{F} = \{u(b) \in \mathbb{R}^2 : b \in B\}$. Player l's minmax payoff is $\underline{v}_l^p \equiv \min_{b_{-l}\in A_{-l}} \max_{b_l \in A_l} u_l(b_l, b_{-l})$ and the set of all strictly individually rational payoffs is given by $\mathcal{F}^p = \{v \in \mathcal{F} : v_l > \underline{v}_l^p, l = i, j\}$. Following Table 1, $\underline{v}_l^p = P$ and $\mathcal{F}^p = \{v \in \mathcal{F} : v_l > P, l = i, j\}$.

Table 1
The two-player prisoners' dilemma.

		Player j	
		C	D
Player i	C	R, R	S, T
	D	T, S	P, P

$T > R > P > S$ and $R > \frac{T+S}{2}$.