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Compact operator method of accuracy two in time and four in space for the numerical solution of coupled viscous Burgers' equations ☆

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ABSTRACT

In this paper, we propose a new two-level implicit compact operator method of order two in time (t) and four in space (x) for the solution of time dependent coupled viscous Burgers' equations. In this method, we did not use any transformation or linearization technique to handle nonlinearity. We use only 3-spatial grid points and the obtained tridiagonal nonlinear system has been solved by Newton's iterative method. The test problems considered in the literature have been discussed to demonstrate the strength and utility of the proposed method. The computed numerical solutions are in good agreement with the exact solutions and competent with those available in earlier studies. We show that the proposed method enables us to obtain high accurate solution for high Reynolds number.

1. Introduction

We consider the numerical solution of coupled viscous nonlinear Burgers' equation (see [1]) of the form:

$$\varepsilon\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}+\alpha_1 u\frac{\partial u}{\partial x}+\alpha_2\frac{\partial(uv)}{\partial x},\quad a<x<b,\quad t>0 \tag{1.1}$$

$$\varepsilon\frac{\partial^2 v}{\partial x^2}=\frac{\partial v}{\partial t}+\beta_1 v\frac{\partial v}{\partial x}+\beta_2\frac{\partial(uv)}{\partial x},\quad a<x<b,\quad t>0 \tag{1.2}$$

with the initial conditions

$$u(x,0)=f_0(x),\quad v(x,0)=g_0(x),\quad a\leqslant x\leqslant b \tag{2}$$

and boundary conditions

$$u(a,t)=f_1(t),\quad u(b,t)=f_2(t),\quad t\geqslant 0 \tag{3.1}$$

$$v(a,t)=g_1(t),\quad v(b,t)=g_2(t),\quad t\geqslant 0 \tag{3.2}$$

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where $0 < \varepsilon \ll 1$ is viscosity, $R_e = \varepsilon^{-1} > 0$ is Reynolds number, α_1 and β_1 are real constants, α_2 and β_2 are arbitrary constants depending on the system parameters (see [2]). The coupled Burgers' equations (1.1) and (1.2) represent system of one space dimensional nonlinear parabolic equations with two unknown variables u and v. A great amount of research work has been carried out in the past for the numerical solution of nonlinear parabolic equations. The Crank–Nicolson type scheme has enjoyed great popularity which is based on the classical trapezoidal formula for integration in time and the central difference formula for spatial discretization for the solution of nonlinear parabolic equations. Lower order convergent numerical methods for the solution of parabolic equations have been discussed by Douglas and Jones [3], Reynolds [4], and Ciment et al. [5]. Dai and Chen [6] have developed a lower order conditionally stable explicit scheme for viscous Burgers' equation. Using uniform mesh discretization and 3-spatial grid-points, two-level implicit difference methods of order two in time (t) and four in space (x) for the solution of viscous Burgers' equation in one dependent variable were discussed in [7–13]. The coupled Burgers' equations belong to an important class of fluid flow equations. Mathematical models of Burgers' equations are described in [14–17]. Wei and Gu [18] have proposed conjugate filter approach to solve scalar Burgers' equation. Abdou and Soliman [19] have presented variational iterative method for Burgers' equations. Adomian–Pade technique has been used by Dehghan et al. [20] to solve these equations. Siraj-ul-Islam [21] has developed a mesh free interpolation method for the solution of coupled nonlinear partial differential equations. A Fourier pseudospectral method has been proposed by Rashid and Ismail [22] to solve coupled Burgers' equations. Kaya [23] has used Adomian decomposition method and obtained an explicit solution of coupled viscous Burgers' equation in the form of a convergent power series with easily computable components. Soliman [24] has used the extended tanh-function method to solve Burgers' type equations. Recently, Mittal and Arora [25], and Mittal and Jain [26] have solved the coupled Burgers' equation by using collocation of B-spline and cubic B-spline functions. Siraj-ul-Islam et al. [27] have obtained the numerical solution of the transient coupled Burgers' equations by collocation of radial basis functions. Using Differential quadrature method, Mittal and Jiwari [28] have solved coupled Burgers' equation. To our knowledge, no numerical method of order two in time (t) and four in space (x) for the coupled Burgers' equations (1.1) and (1.2) has been developed so far. In this article, we present a new two-level implicit compact operator method of accuracy two in time and four in space for the solution of coupled viscous Burgers' equations (1.1) and (1.2). In particular, we use only three spatial grid points and do not require any fictitious point for computation. The numerical solution has been computed without transforming the equations and without using linearization. The paper is arranged as follows: In Section 2, we describe the operator compact method. In Section 3, we give the derivation of the method. In Section 4, we discuss the Newton iteration procedure for implementing the proposed method for the Eqs. (1.1) and (1.2). In Section 5, numerical results are presented for various test problems with tabular and graphical illustrations. Conclusions are given in Section 6.

2. Description of the operator compact method

We may re-write the differential equations (1.1) and (1.2) as

$$\varepsilon \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} + (\alpha_1 u + \alpha_2 v)\frac{\partial u}{\partial x} + \alpha_2 u \frac{\partial v}{\partial x}, \quad a < x < b, \quad t > 0, \tag{4.1}$$

$$\varepsilon \frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t} + \beta_2 v \frac{\partial u}{\partial x} + (\beta_1 v + \beta_2 u)\frac{\partial v}{\partial x}, \quad a < x < b, \quad t > 0. \tag{4.2}$$

We assume that $u(x,t) \in C^6$ in the solution region $\Omega \equiv \{(x,t) : a < x < b, t > 0\}$. Let $h > 0$ and $\tau > 0$ be the mesh spacing in the space and time directions, respectively. Let us consider the interval $[a,b]$ with the uniform partition $a = x_0 < x_1 < x_2 < \ldots < x_N < x_{N+1} = b$, where $x_l = a + lh$, $l = 0(1)N+1$ are called knots, $b - a = (N+1)h$ and $t_j = j\tau$, $0 < j < J$, N and J being positive integers. Let $\lambda = (\tau/h^2) > 0$ be the mesh ratio parameter. We replace the region Ω by a set of grid points (x_l, t_j). Let u_l^j and v_l^j be the discrete solutions of $u(x,t)$ and $v(x,t)$ at the grid point (x_l, t_j). Let U_l^j and V_l^j be the exact solution values of $u(x,t)$ and $v(x,t)$ at the same grid point (x_l, t_j).

At the grid point (x_l, t_j), the given differential equations (4.1) and (4.2) may be written as

$$\varepsilon u_{xxl}^j = u_{tl}^j + (\alpha_1 u_l^j + \alpha_2 v_l^j) u_{xl}^j + \alpha_2 u_l^j v_{xl}^j \equiv f_l^j, \tag{5.1}$$

$$\varepsilon v_{xxl}^j = v_{tl}^j + \beta_2 v_l^j u_{xl}^j + \left(\beta_1 v_l^j + \beta_2 u_l^j\right) v_{xl}^j \equiv g_l^j. \tag{5.2}$$

We define the following operators:

$$\delta_x u_l^j = u_{l+\frac{1}{2}}^j - u_{l-\frac{1}{2}}^j, \quad \mu_x u_l^j = \frac{1}{2}\left(u_{l+\frac{1}{2}}^j + u_{l-\frac{1}{2}}^j\right), \tag{6.1}$$

$$\delta_t u_l^j = u_l^{j+\frac{1}{2}} - u_l^{j-\frac{1}{2}}, \quad \mu_t u_l^j = \frac{1}{2}\left(u_l^{j+\frac{1}{2}} + u_l^{j-\frac{1}{2}}\right), \tag{6.2}$$

where δ_x and μ_x are second-order central difference and averaging operators with respect to x-direction, respectively, etc.