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A new approach to grid search method in slope stability analysis using Box–Behnken statistical design

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ABSTRACT

Grid search method for locating the critical failure surface is extended by deriving additional analytical expressions for slip center grid ($x_{\min}$, $z_{\min}$; $x_{\max}$, $z_{\max}$), where global minimum of safety factor occurs, including the prediction of minimum and maximum values for safety factor ($Fs_{\min,\max}$) and for slip circle radius ($R_{\min,\max}$). Derived models are proposed in a form of nonlinear functions of geometrical parameters (slope height H, depth to bedrock d and slope angle β) and soil factors (bulk density γ, cohesion c, angle of internal friction φ and pore water pressure coefficient r_u). Research was performed using Box–Behnken experimental design, for which the input data were provided by Spencer limit equilibrium analyzes of different slopes with circular slip surface. Reasonable predictive power of the proposed models was verified both by internal and external validation, latter of which included the analyzes of slopes with random geometrical and soil properties. Regarding the impact of input parameters, β has the strongest influence on response values (Fs, R, x, z), with the predominant linear and quadratic effect. As for the influence of remaining factors, c and φ also have strong impact on Fs, while H and φ have significant influence on slip circle radius and the location of slip center grid. However, due to existence of two-factor interactions, it is shown that the effect of β on Fs, x, z and R is highly dependent on the values of c, φ, r_u, H and d/H, including the significant effect of $r_u \times \varphi$, $c \times H$ and $c \times \gamma$. When compared to traditional grid search method, proposed approach could be used to locate the circular slip surface with global minimum of safety factor, without the need for additional slope stability analyzes.

1. Introduction

Quick estimation of slope stability has an important role in geotechnical design, especially in case of limited amount of time for geotechnical report to be prepared or when fast calculation is demanded in field conditions. In order to evaluate the safety factor (Fs) and to locate the position of critical slip surface in a short computational time, applied methods should meet certain requirements: they should be user-friendly, based on rather graphical than numerical approach, and have to provide a high level of reliability and accuracy, especially regarding the slopes at the limit of stability. These demands are certainly fulfilled when using modern numerical methods, such as nature-inspired algorithms [1–6], which typically include

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different optimization techniques, artificial neural networks or methods based on genetic algorithms. These techniques are widely being used due to their ability to easily establish and describe the complex nonlinear relations among the interacting units. Genetic algorithms are particularly favored for solving global optimization problems, especially when standard optimization procedures are not expected to provide reliable results, as is the case in present research. However, the main disadvantage of all these numerical methods lies in the fact that they could not be applied manually without the strong processing unit, which is of primary importance for a fast preview of slope stability. Therefore, when quick analysis is needed, classical methods are still being widely used in engineering practice, including the application of stability charts or the combination of different circular surface search methods and limit equilibrium analyzes.

Employment of stability charts, originally proposed by Taylor [7] and Janbu [8], enables calculation of slope safety factor as a function of geometrical slope characteristics and geomechanical properties, including the location of critical slip surface. In that way, Taylor's and Janbu's charts provide a whole picture of slope stability, and their use in engineering practice has given sufficiently accurate and reliable results so far. Besides these charts, there are only few attempts to further develop such an approach that enables location of critical slip surface, with corresponding slope safety factor [9]. Modern studies are usually focused on developing methods that provide only the value of slope safety factor for *a priori* determined slip surface [10–13].

Regarding the circular surface search methods, like grid search method [14], slope search or auto refine search method [15], they enable locating the critical slip surface for which the safety factor is later determined by using some of the conventional techniques, like limit equilibrium analyzes. These search methods are widely used in engineering practice, which is confirmed by their implementation in modern software packages [15]. The latter two methods, slope search and auto refine search, are routinely applied within the existing software solutions, since they use path search technique based on generating numerous possible slip surfaces, for which the safety factor is calculated using some of the traditional limit equilibrium analyzes [15]. Typically, these methods are successful in finding the global minimum of safety factor, which recently became possible with powerful optimization procedures based on Monte Carlo technique [16]. On the other hand, unlike slope search or auto refine search method, grid search method can be applied manually, which makes it suitable for a quick preview of slope stability. Nevertheless, its main disadvantage lies in the fact that it does not always provide a global minimum of safety factor, which is especially important for "critical" slopes with low values of safety factor. Within this research, we aim to extend the existing grid search method, by developing explicit means for locating the critical slip surface where the function of safety factor reaches its global minimum.

Original grid search method [14] assumes that center of critical slip circle falls into a rectangular area, whose position above the slope surface is determined on the basis of solid engineering experience. This rectangular area represents a slip center grid, where each point is a center of a potential critical slip surface. For a certain radius range, slope stability is examined for each point until a minimum value of Fs is found. Although this method is very intuitive and easy applicable in practice, there is no guarantee that safety factor for the obtained critical slip surface has reached its global minimum. Such deficiency of grid search method could lead to wrong interpretations of slope stability, especially when slope is at the verge of stability ($Fs \approx 1$). In order to eliminate such a possibility, we intend to develop a method for precise location of the slip center grid, where the function of safety factor reaches its global minimum with high statistical reliability. In particular, the main goal of the present research is to extend the existing grid search method by providing explicit mathematical expression for coordinates of slip center grid, including the ranges of slip circle radii and safety factors for slip circles whose centers are within the previously determined grid. For this purpose, we employ the method of Box–Behnken statistical design [17,18], which was not previously applied in the area of geotechnical analysis, even though it has already given reliable results in analytical chemistry [19,20] and concrete production [21,22].

Structure of the paper is as follows. In Section 2 we briefly describe the applied methods, while proposed models are given in Section 3. Verification of the derived equations is performed in Section 4. Impact of individual input factors, as well as their co-action, is described in Section 5, while discussion on the obtained results and main conclusions are given in the last section, including suggestions for further research.

2. Applied methods

In order to obtain reliable mathematical expressions for locating the slip center grid, including the corresponding critical slip surfaces and safety factors, we employ Box–Behnken statistical design, which is based on a small number of slope stability analyzes with such combination of input factors that provide responses of the highest possible statistical accuracy [23]. For that purpose, appropriate values of all input factors are defined in a way that they uniformly fill the experimental space determined by the range of input parameters. For such values, critical slip surface with the lowest safety factor is located using SLIDE® Rocscience [15]. In particular, possible slip surfaces are determined using auto refine search method, with minimum depth of slip surface at 1 m, so as to avoid the occurrence of slip circles at the slope surface. For convenience, it was assumed the soil failure occurs according to Mohr–Coulomb criterion. Slope safety factors are calculated by applying Spencer limit equilibrium method, which is chosen as the simplest method that satisfies all the equilibrium conditions [24]. Results of the series of stability analyzes, which are performed according to the pre-defined statistical design, are further used to establish relations among the response values (safety factor, slip circle radius and coordinates of slip center grid) and examined input geometrical and geomechanical parameters. Such relations or models are typically defined using multiple