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Multi-stage stochastic mean–semivariance–CVaR portfolio optimization under transaction costs

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ABSTRACT

In this paper, we present a model for portfolio selection, characterized on the basis of three parameters: the expected value, semivariance, and Conditional Value-at-Risk (CVaR) at a specified confidence level. In order to solve the proposed model, we design a hybrid of genetic algorithm (GA) and particle swarm optimization (PSO) algorithm. Because the effectiveness of meta-heuristic algorithms significantly depends on the proper choice of parameters, a Taguchi experimental design method is applied to set the suitable values of parameters to improve the hybrid algorithm performance. Finally, some numerical examples are given to illustrate the effectiveness of the proposed algorithm.

1. Introduction

Portfolio selection problem is a classical financial problem introduced by Markowitz and includes two integral parts, risk and return [1]. The main aim of this problem is to maximize the expected return for a certain level of risk or minimize the risk for a certain return. This model established a fundamental base for single-period portfolio selection. After proposing the mean–variance model, many extensions, inspired by that model, have been proposed such as [2–5].

In the real world, the portfolio strategies are usually multi-period because the investor is able to rebalance his portfolio in each time period. Rebalancing the portfolio, the investor incurs transaction costs. In order to be more realistic, transaction costs, which is one of the main concerns of the today's portfolio managers, should be taken into account.

Multi-period problems as general cases where an investor can make a sequence of decisions impacting each other and the objective is to find, at each time period, the optimal allocation decision wherein it tries to maximize the expected utility of wealth. It can be defined as maximization of the return value and minimization of the risk value at the final period of time. Many financial applications, such as asset-liability management, index tracking, and investment management, can be represented as a sequence of decisions.

Several authors investigated on Multi-period portfolio selection models. Li and Ng [6] proposed an analytical optimal solution to the multi-period mean–variance formulation. Wei and Ye [7] developed a multi-period mean–variance portfolio selection model, taken bankruptcy constraint into consideration, in stochastic market. Also, a multi-period mean–variance optimization, entailing the construction of scenario tree is proposed by Gülpınar and Rustem [8] to present uncertainties and associated possibilities in future stages. Çelikyurt and Özekici [9] introduced several multi-period portfolio optimization models in stochastic market using the mean–variance approach. Calafiore [10] represented a multi-period optimization with linear control policies as he concerned with multi-period sequential decision problems for financial asset allocation.

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Multi-period portfolio optimization has been solved via Conditional Value-at-Risk (CVaR) under nonlinear transaction costs in [11]. Sadjadi et al. [12] constructed a fuzzy multi-period portfolio by taking different borrowing and lending rates into consideration. Multi-stage portfolio optimization problem solved with drift particle swarm optimization and results were superior as well as more efficacious than other tested algorithms in [13]. Liu et al. [14] represented a mean-skewness multi-period portfolio in fuzzy environment and pointed out that it performs better in comparison with those which do not contain the skewness as a risk measure.

Researches have showed that stochastic programming models are pliable means to describe financial optimization problems under uncertainty. Various formulations for the multi-stage financial problem have been proposed in the literature (e.g. [15–17]). Carino et al. [18] formulated an asset/liability management problem using multi-stage stochastic programming. A hybrid simulation/tree multi-period stochastic programming model has been constructed for optimal asset allocation in [19].

As mentioned above, researchers often used variance as a risk measure. Because of measuring the actual investment risk, asymmetric return distributions, and need of investor; downside risk measures, such as semivariance, Value-at-Risk (VaR), or Conditional Value-at-Risk (CVaR) should be replaced with variance. Describing the loss, occurring over a given period, at a given confidence level, VaR can be one of the possible choices in risk management. Baixauli-Soler et al. [20] proposed a mean-VaR portfolio selection model under real constraints and solved it by a multi-objective genetic algorithm. However, Szego [21] showed that VaR was not an acceptable and correct risk measure due to the following reasons: it did not measure losses exceeding VaR, VaR was not sub-additive. Having understood VaR's imperfections, CVaR, which is defined as the expected value of the losses exceeding VaR, can be adopted rather than VaR. It is worth noting that there are several worthwhile properties in computational and theoretical aspects for CVaR as a risk measure [22,23].

Downside-risk measure is used as a risk measure to study the multi-period portfolio selection problem [24]. A multi-period mean–semivariance portfolio selection model is proposed and variance is substituted by semivariance [25,26]. Zhang and Zhang [27] constructed a new multi-period stochastic optimization based on CVaR and took transaction costs into consideration. Zhang et al. [28] proposed a mean–semivariance model for multi-period portfolio selection in fuzzy environment. Arnot and Wagner [29] showed that neglecting transaction costs would result in a disappointing performance; hence, transaction costs are becoming one of the main concerns of today's portfolio managers. Transaction costs have been taken into consideration in many multi-period portfolio selection models, for instance see [11,27,28,30,31].

There have been a lot of researches on multi-period portfolio selection problem, while most of them used one risk measure and specially variance. In this paper, we use semivariance rather than variance. In addition, we consider CVaR as a risk measure which can be complementary to semivariance. We also take transaction costs into account; thus we give a new perspective on the multi-stage stochastic problems. Moreover, we design a hybrid of genetic algorithm and particle swarm optimization. Since effectiveness of the algorithms depends on the correct choice of parameters, we also use Taguchi method for parameter tuning of the algorithm.

The rest of the paper is organized as follows. Section 2 presents the conception and formulation of the multi-stage stochastic mean–semivariance–CVaR model. Then, we convert it into a single objective model by using the e-constrained method. In Section 3, we design a hybrid algorithm for solving the proposed model. We tune the parameters of the proposed algorithm using Taguchi method in Section 4. Numerical examples and ranking of algorithms with TOPSIS are provided in Section 5. Finally, conclusions are drawn in Section 6.

2. Multi-stage portfolio optimization model

2.1. Problem statement

Multi-stage portfolio optimization is considered as a multi-period dynamic problem where transactions take place at discrete time points. Multi-stage portfolio problem can be defined as follows: there are N risky assets, one riskless asset (asset 0). A planning horizon consists of T stages, moments in time when decisions are taken. Time intervals can be varied from minutes to years and decisions are made at the beginning of the stages. The first stage represents the current date. Proceeds from the sales are added to and expenses from the purchases are detracted from the cash account, asset 0. At time $t+1$, based on the realized returns over $(t, t+1]$ the investor's holdings are updated. For simplicity, we will assume that returns of the riskless asset for lending (r_l) and borrowing (r_b) are fixed. The date of the planning horizon generally depicts a point at which the investor has some critical restrictions, such as the repayment of an overwhelming liability. At the end of time period T, an investor collects his final wealth W_T. The investor's goal is to manage a portfolio of assets to maximize the expected utility of final wealth $E[U(W_T)]$.

Uncertainty is modeled through scenarios and each scenario describes a possible realization of all uncertain parameters in the model. Each scenario $\bar{S}$ has a probability p_s, where $p_s > 0$ and $\sum_{s=1}^{S} p_s = 1$. For simplicity, we supposed that scenarios are equally possible to happen; thus $p_s = \frac{1}{S}$.

In a dynamic model, an appropriate way to represent uncertainty is scenario tree because information appears on actual value of the uncertain parameters in stages. Scenario is a path from the root to a leaf, shown in Fig. 1. Any node of the tree, corresponding to time t, stands for a possible state of the world and it is crystal clear that all scenarios, passing these nodes, have the same history in periods $0, 1, 2, \ldots, T-1$.