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Computation of vibration solution for functionally graded carbon nanotube-reinforced composite thick plates resting on elastic foundations using the element-free IMLS-Ritz method

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ABSTRACT

This paper explores the element-free IMLS-Ritz method for computation of vibration solution of thick functionally graded carbon nanotube-reinforced composite (FG-CNTRC) plates resting on elastic foundations. The shear deformation effect is incorporated through the first-order shear deformation theory (FSDT). The cubic spline weight function and linear basis are utilized in the approximation. Regular node arrangements and cell background meshes are employed in the numerical integration. The penalty method is adopted to impose the essential boundary conditions. Numerical stability and applicability of the IMLS-Ritz method are examined through solving a few numerical example problems. The influence of Winkler modulus parameters on the vibration behavior of FG-CNTRC plates is studied. Besides, the effects of CNT volume fraction, CNT distribution, plate thickness-to-width ratio, plate aspect ratio on FG-CNTRC plates are investigated under different boundary conditions. The vibration frequencies and mode shapes of the FG-CNTRC plates on different Winkler foundations are presented.

1. Introduction

Thick plate is three-dimensional in nature. Basically it should be treated as a three-dimensional problem. To avoid complicated mathematics in three-dimensional analysis and taking the advantages of being able to treat it as a two-dimensional problem, numerous two-dimensional plate theories have been developed. It began with the Kirchhoff–Love theory for thin plate analysis [1]. For thick plates, Reissner [2] considered the secondary effects, i.e. transverse shear deformation, showing substantial impact in bending analysis. Extending Reissner's idea, Mindlin [3] proposed a theory for vibration analysis of thick plates, including the effects of transverse shear deformation and rotary inertia. This plate theory has later been referred to as the Reissner–Mindlin plate theory or simply the first-order shear deformation theory (FSDT).

Under harmonic vibration, the governing differential equations based on the FSDT for a thick plate resting on an elastic foundation can be expressed in the following forms

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$$KGh\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}+\theta_x\right)+\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}+\theta_y\right)\right]+(\rho h\omega^2-k)w=0, \quad (1)$$

$$D\left[\frac{\partial}{\partial x}\left(\frac{\partial \theta_x}{\partial x}+\nu\frac{\partial \theta_y}{\partial y}\right)\right]+\frac{(1-\nu)D}{2}\left[\frac{\partial}{\partial y}\left(\frac{\partial \theta_y}{\partial x}+\frac{\partial \theta_x}{\partial y}\right)\right]-KGh\left(\frac{\partial w}{\partial x}+\theta_x\right)+\frac{\rho h^3}{12}\omega^2\theta_x=0, \quad (2)$$

$$D\left[\frac{\partial}{\partial y}\left(\frac{\partial \theta_y}{\partial y}+\nu\frac{\partial \theta_x}{\partial x}\right)\right]+\frac{(1-\nu)D}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \theta_y}{\partial x}+\frac{\partial \theta_x}{\partial y}\right)\right]-KGh\left(\frac{\partial w}{\partial y}+\theta_y\right)+\frac{\rho h^3}{12}\omega^2\theta_y=0, \quad (3)$$

where K is the shear correction factor, $D=Eh^3/[12(1-\nu^2)]$ is the flexural rigidity of plate, ρ is the mass density of plate, ω is the plate vibration frequency, w is the transverse displacement, and θ_x and θ_y are the rotations in the y and x directions.

Apparently, for solving the above governing differential equations, solution to the boundary value problems can be approximated using any traditionally discretization techniques, for examples, the finite element method [4], the finite difference method [5], the differential quadrature method [6,7], and other methods [8–11]. In recent years, a class of new approximate technique, the element-free or mesh-free method, has become a powerful numerical tool for locating approximate solutions for such boundary value problems. The element/mesh-free method furnishes solution with: (i) only a minimum of meshing or no meshing at all, and (ii) a set of scattered nodes are used instead of meshing the problem domain [12]. Numerous element/mesh-free methods have been developed based on different shape functions, such as the element-free Galerkin method [13], the improved element-free Galerkin (IEFG) method [14–16], the smooth particle hydrodynamics methods [17], the element-free kp-Ritz method [18–22], the element-free IMLS-Ritz method [23,24], the DSC-Ritz method [25–27], the meshless local Petrov–Galerkin method [28], and the local Kriging meshless method [29,30].

In particular, the DSC-Ritz method [25,26] was recently applied for vibration analysis of Mindlin plates. In [25], the work takes advantage of both the local bases of the DSC algorithm and the pb-2 Ritz boundary functions to arrive at a new numerical approach, called the DSC-Ritz method where two-basis functions are constructed by using DSC delta sequence kernels of the positive type. The energy functional of the Mindlin plate was represented by the newly constructed basis functions and minimized via the Ritz variational principle. In [26], the vibration analysis of thick shallow shells vibrating at very high modes was investigated using the discrete DSC wavelet kernels of the Dirichlet type and the Ritz method. The method is numerically very efficient because it does not encounter numerical instability even at the 1500th mode using a reasonable number of grid points. Because of the very high numerical accuracy, the DSC-Ritz method was used to predict the missing modes when using the exact frequency relationship between Kirchhoff and Mindlin plates [27]. In [27], it was shown that the use of the Kirchhoff–Mindlin plate relationship leads to missing vibration modes due to the lack of consideration for the transverse shear modes and the coupled bending-shear modes in the relationship. The missing modes appear at the relatively high order modes and an efficient state-space technique was also used to confirm the prediction.

The element-free IMLS-Ritz method [23,24] was successfully employed for solving many mechanical and mathematical problems. The method employs the improved moving least-squares (IMLS) approximation for construction of its shape functions in the element-free procedure, overcoming the drawback of the MLS method. Under the IMLS approximation, the basic function is formed by an orthogonal function system with a weight function. Unlike the MLS method, the algebra equation system of the IMLS approximation is well-conditioned which can be solved without obtaining the inverse matrix. Apart from this advantage, the problem can be modelled with fewer coefficients in the IMLS approximation than that in the MLS approximation, leading to a higher computational efficiency. This paper will employ the IMLS-Ritz method to study the vibration of FG-CNTRC thick plates resting on elastic foundations. In the existing literature, there are some notable works reported on the bending, buckling, vibration, and large deformation of FG-CNTRCs beams, rectangular plates and shells [31–39]. Most of the earlier works on FG-CNTRC plates have been summarized in a recent review article published by Liew et al. [40].

In this study, the element-free IMLS-Ritz method is further explored for vibration analysis of FG-CNTRC thick plates resting on an elastic foundation. Unlike the DSC-Ritz method [25,26], the IMLS-Ritz method employs the IMLS approximation based on the orthogonal function system with a weight function as the basis function [23,24]. Using the IMLS-Ritz method, the governing eigenvalue equation is derived. The numerical procedure involves employing the cubic spline weight function and linear basis in its approximation. The regular node arrangements and background mesh of cells are used in the numerical integration. The penalty method is employed to impose the essential boundary conditions. The vibration frequencies and mode shapes of the FG-CNTRC plates are obtained by solving the governing eigenvalue equation. A few example problems are carefully selected for representing different properties of the FG-CNTRC plates. The numerical stability and applicability of the IMLS-Ritz method are examined. The accuracy is established by comparing the IMLS-Ritz results with the existing solutions. The influence of Winkler modulus parameters on the vibration behavior of the FG-CNTRC plates is studied. Besides, the effects of CNT volume fraction, CNT distribution, plate thickness-to-width ratio, plate aspect ratio on the FG-CNTRC plates are examined under different boundary conditions.

2. Problem statement

The geometry of the FG-CNTRC plates and distributions of CNTs along the thickness direction of the plates, as shown in Fig. 1, are assumed to be