Applied Mathematics and Computation 256 (2015) 505–513

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A fuzzy portfolio selection model with background risk

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ARTICLE INFO

Keywords:
Background risk
Fuzzy number
Possibilistic mean
Possibilistic variance

ABSTRACT

In financial markets, the presence of background risk may affect investors' investments. This article develops a fuzzy portfolio selection model with background risk, based on the definitions of the possibilistic return and possibilistic risk. For the returns of assets obey LR-type possibility distribution, we propose a specific portfolio selection model with background risk. Then, a numerical study is carried out by using the data concerning some stocks. Based on the data, we obtain the efficient frontier of the possibilistic portfolio with background risk, and compare it with the efficient frontier of the portfolio without background risk. Finally, we conclude that the background risk can better reflect the investment risk of the real economy environment which make the investors choose a more suitable portfolio to them.

1. Introduction

Markowitz [1] proposed a mean-variance model for portfolio selection in the earliest stage. Over the last 50 years, his theory has played an important role in the development of modern portfolio selection theory. In Markowitz's portfolio theory, it is assumed that all the investors are averse to risk, asset returns are random variables, the expected return of a portfolio is defined by the mean, and the risk is characterized by the variance. The objective of an investor is to minimize the variance of a portfolio for a fixed expected return. Therefore, the portfolio selection problem can be stated as follows:

$$\begin{cases} \min \quad V(X) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j, \\ s.t. \quad \sum_{i=1}^{n}\mu_i x_i \geqslant \bar{r}, \\ \qquad \sum_{i=1}^{n} x_i = 1, \\ \qquad i = 1, 2, \ldots, n, \end{cases}$$

where $\bar{r}$ denotes a required return of the portfolio, σ_{ij} is the covariance between assets i and j, μ_i is the expected return rate of an asset i, and x_i is the weight of an asset i.

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http://dx.doi.org/10.1016/j.amc.2015.01.007
0096-3003/

Researches on the mean-variance portfolio selection problem typically include Sharpe [2], Merton [3, Perold [4], Pang [5] etc. However, because of the complexity of financial systems, in several cases the input data is not precise but fuzzy. Therefore the decision maker should not consider parameters (goals and constraints) using crisp numbers or unique distribution functions, but use fuzzy numbers or fuzzy probability distribution functions [6]. Possibility theory was proposed by Zadeh [7] and was advanced by Dubois and Prade [8]. In Zadeh's theory, fuzzy variables are associated with possibility distributions, which is in the similar way that random variables are associated with probability distribution. Carlsson and Fullér [9,10] introduced the notions of possibilistic mean, possibilistic variance and covariance of fuzzy numbers. Georgescu [11] presented an approach to some topics of risk theory in the context of Zadeh's possibility theory. Inuiguchi and Tanino [12] introduced a possibilistic programming approach to the portfolio selection problem based on the minimax regret criterion. Zhang [13] and Zhang et al. [14] discussed the portfolio selection problems based on the lower, upper and crisp possibilistic means and possibilistic variances when short sales were not allowed on all risky assets. Huang [15] discussed fuzzy chance-constrained portfolio selection.

However, most of the above-mentioned papers do not address the issue of background risk because, under the assumption of complete markets, background risk can be priced and capitalized into wealth [16]. The statistical properties of background risk are then irrelevant to the allocation of wealth between risky stocks and risk-free bonds [16]. In practice, investors often face background risk such as those arising from labor income and real estate which are not insurable in financial markets. Heaton and Lucas [16] focused on how the presence of background risk influenced portfolio allocation. Eeckhoudt et al. [17] analyzed the effect of background risk. Gollier [18] analyzed risk vulnerability and the tempering effect of background risk. Tsanakas [19] constructed a distortion-type risk measure which evaluated the risk of any uncertain position in the context of a portfolio with a fixed background risk. Baptista [20] proposed the optimal delegated portfolio management with background risk. Eichner and Wagener [21] analyzed the effects on portfolio composition of changes in the mean, the variance and the covariance between assets and background risk, and studied the tempering effect of increasing background risk. Georgescu [22,23] proposed the investment models with background risk combining probabilistic and possibilistic aspects. Cardak and Wilkins [24] studied the portfolio allocation decisions of Australian households, and analyzed the effect of labor income uncertainty and health risk. Jiang et al. [25] investigated the impact of background risk on an investor's portfolio choice, and analyzed the investor's hedging behavior in the presence of background risk. Alghalith [26] introduced an incomplete-market dynamic investment model with a correlated background risk. Menoncin [27] analyzed the portfolio problem with the stochastic investment opportunities and background risk.

Although a considerable number of research papers have been published for portfolio selection problem in fuzzy environment, there is few research on portfolio selection problem with background risk based on possibility theory. The purpose of this paper is to discuss a fuzzy portfolio with background risk based on possibilistic theory. Different from Zhang [13,14], we discuss a possibilistic portfolio selection model with background risk by assuming the returns of assets obey LR-type possibility distributions. We consider constraints on holdings of assets in the portfolio model. Finally, we carry out a study by using the data concerning some stocks. We conclude that the investors are able to choose a more suitable portfolio for them with background risk.

The rest of the paper is organized as follows. In Section 2, we introduce indicators of fuzzy numbers and their properties. In Section 3, we propose a fuzzy portfolio selection model with background risk. Assuming that the expected rate of returns is a LR-type distribution fuzzy variable, we derive a crisp equivalent form of the possibilistic portfolio with background risk. In Section 4, a numerical example is given to illustrate our proposed effective approaches. Finally, Section 5 presents our conclusions.

2. Indicators of fuzzy numbers

Let us introduce some definitions, which will be used in the following section. A fuzzy number $\widetilde{A}$ is a fuzzy set of the real line $\mathcal{R}$ with a normal, fuzzy convex and continuous membership function of bounded support. The family of fuzzy numbers is denoted by $\mathcal{F}$. Let A be a fuzzy number with γ-level set $[\widetilde{A}]^{\gamma} = [a_1(\gamma), a_2(\gamma)](\gamma > 0)$. In 2001, Carlsson and Fullér [28] defined the upper and lower possibilistic mean values of a fuzzy number $\widetilde{A}$ as

$$M_U(\widetilde{A}) = 2\int_0^1 \gamma a_2(\gamma)d\gamma, \tag{1}$$

and

$$M_L(\widetilde{A}) = 2\int_0^1 \gamma a_1(\gamma)d\gamma. \tag{2}$$

Carlsson and Fullér [28] also introduced the crisp possibilistic mean value of a fuzzy number as

$$\overline{M}(\widetilde{A}) = \int_0^1 \gamma(a_1(\gamma) + a_2(\gamma))d\gamma = \frac{M_U(\widetilde{A}) + M_L(\widetilde{A})}{2}. \tag{3}$$

The crisp possibilistic mean value of $\widetilde{A}$ is the arithmetic mean of its lower possibilistic and upper possibilistic mean values.