Applied Mathematics and Computation 256 (2015) 514–528

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Adaptive fuzzy tracking control for stochastic nonlinear systems with unknown time-varying delays ☆

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ARTICLE INFO

Keywords:
Fuzzy logic systems (FLS)
Stochastic nonlinear systems
Time-varying delays
Backstepping
Lyapunov–Krasovskii functionals

ABSTRACT

This paper addresses the problem of adaptive tracking control for a class of stochastic strict-feedback nonlinear time-varying delays systems using fuzzy logic systems (FLS). In this paper, quadratic functions are used as Lyapunov functions to analyze the stability of systems, other than the fourth moment approach proposed by H. Deng and M. Krstic, and the hyperbolic tangent functions are introduced to deal with the Hessian terms. This approach overcomes the drawback of the traditional quadratic moment approach and reduce the complexity of design procedure and controller. Based on the backstepping technique, the appropriate Lyapunov–Krasovskii functionals and the FLS, the adaptive fuzzy controller is well designed. The proposed adaptive fuzzy controller guarantees that all the signals in the closed-loop system are bounded in probability and the tracking error can converge to a small residual set around the origin in the mean square sense.

1. Introduction

It is well known that stochastic disturbances often exist in many practical systems and their existence is a source of instability of control systems. Thus, the investigation on stability analysis and control design of stochastic systems has received increasing attention in the past decade, such as [1,4–7,10,12,16–18,23,28–35] and references therein. In particular, for a class of stochastic nonlinear strict-feedback (or output-feedback) systems, many interesting control schemes [4,5,18,23] have been proposed by using the well-known backstepping technique. [6] first derived a backstepping design approach for stochastic nonlinear strict- feedback systems motivated by a risk-sensitive cost criterion. Since then, a series of extensions has been made under different assumptions or for different systems [7,12]. By using the quartic Lyapunov functions instead of the classical quadratic functions [4] solved the (adaptive) stabilization problem of stochastic strict-feedback (or output-feedback) systems, and then, this design idea was extended to several different cases, such as tracking control [23,28,30,35], decentralized control [10,16] and control of high-order systems [5,17,32–34]. It is well known that the classical quadratic Lyapunov functions are usually used to deal with the tracking control problem of the deterministic nonlinear systems [2,3]. However, up to now to the authors' knowledge, there are only several literatures using the classical quadratic Lyapunov functions to deal with the stochastic tracking control problem, but they are all with respect to a risk-sensitive cost criterion,

☆ This work is supported by Ph.D. Programs Foundation of Ministry of Education of China (JY0300137002), the Fundamental Research Funds for the Central Universities (JB142001-6), the Youth Foundation of Xi'an University of Architecture and Technology (No. QN1436) and the Talent Foundation of Xi'an University of Architecture and Technology (No. RC1425).
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http://dx.doi.org/10.1016/j.amc.2014.12.104
0096-3003/

such as [12,18]. Thus, if not under the risk-sensitive cost criterion, how to design the stochastic tracking controller by using the quadratic Lyapunov is worth studying.

On the other hand, fuzzy logic systems (FLS) and neural networks (NNs) have been proved to be very useful tools for solving the control problem of uncertain systems. However, compared with NNs, the main advantage of FLS is that it can combine some experience and knowledge from designers or experts [9]. These experience and knowledge can initiate the estimated parameters in order to make them close to their optimal values, which is very important to improve the control transient performance. Therefore, this paper will use the FLS to deal with the uncertain nonlinear systems.

Another challenging problem in real engineering systems is that time delays are frequently encountered, and the existence of time delays usually becomes the source of instability and degrading performance of systems. Therefore, the stability analysis and controller synthesis of nonlinear systems with time-delay are important. With the development of adaptive control and backstepping designs in nonlinear systems [22], recently, for nonlinear systems with constant delays [25,26], several fuzzy (neural) adaptive control schemes have been reported by combining Lyapunov–Krasovskii functionals and the backstepping technique. However, for the nonlinear systems with unknown time-varying delays, how to design the adaptive fuzzy controller by using the backstepping technique and the appropriate Lyapunov–Krasovskii functionals is a challenging subject.

Based on the above observation, in this paper, the problem of output tracking is revisited for stochastic strict-feedback nonlinear systems with unknown time-varying delays using fuzzy control. Under the approximately assumptions, quadratic functions are used as Lyapunov functions to analyze the stability of systems and the hyperbolic tangent functions are introduced to deal with the Hessian terms, while the Lyapunov–Krasovskii functionals are constructed to compensate the unknown time-varying delays terms. Then, the FLS are employed to approximate the unknown nonlinear functions. Finally, the adaptive backstepping approach is utilized to construct the fuzzy controller. The two main advantages of the scheme are that:

(1) In this paper, quadratic functions are used as Lyapunov functions to analyze the stability of the systems and the hyperbolic tangent functions are introduced to deal with the Hessian terms. Thus, the long-standing obstacle for stochastic systems control by quadratic Lyapunov function is overcome.
(2) The developed control scheme can handle stochastic nonlinear systems with time-varying delays.

It can be proven that all the signals in the closed loop system are bounded in probability and the tracking error can converge to a small residual set around the origin in the mean square sense. Simulation results are provided to show the effectiveness of the proposed approach.

2. Preliminaries

2.1. Notations

(1) $C([-d,0];R^n)$ denotes the space of continuous R^n valued functions on $[-d,0]$ endowed with the norm $\|\cdot\|$ defined by $\|f\| = \sup_{x\in[-d,0]}|f(x)|$ for $f \in C([-d,0];R^n)$, and $C^b_{\mathcal{F}_0} \times ([-d,0];R^n)$ denotes the family of all $\mathcal{F}_0$-measurable bounded $C([-d,0];R^n)$-valued random variables $\xi = \xi(\theta) : -d \leqslant \theta \leqslant 0$.
(2) C^i denotes the set of all functions with continuous ith partial derivatives; $C^{2,1}(R^n \times [-d,\infty];R_+)$ denotes the family of all nonnegative functions $V(x,t)$ on $\mathrm{R^n} \times [-\mathrm{d},\infty)$, which are C^2 in x and C^1 in t; and $C^{2,1}$ denotes the family of all functions, which are C^2 in the first argument and C^1 in the second argument.
(3) K denotes the set of all functions: $R_+ \to R_+$, which are continuous, strictly increasing, and vanish at zero; K_∞ denotes the set of all functions that are of class K and unbounded.

2.2. Stochastic Stability

Consider an n-dimensional stochastic time-delay system

$$dx(t) = f(x(t),x(t-d(t)),t)dt + g(x(t),x(t-d(t)),t)d\omega, \quad \forall t \geqslant 0, \tag{1}$$

with initial data $x(\theta) : -d \leqslant \theta \leqslant 0 = \xi \in C^b_{\mathcal{F}_0} \times ([-d,0];R^n)$, where $d(t) : R_+ \to [0,d]$ is a Borel measurable function; $f : R^n \times R^n \times R_+ \to R^n$ and $g : R^n \times R^n \times R_+ \to R^{n\times r}$ are locally Lipschitz; and ω is an r-dimensional standard Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{F_t\}_t \leqslant 0, P)$, with Ω being a sample space, $\mathcal{F}$ being a σ field, $\{F_t\}_t \leqslant 0$ being a filtration, and P being a probability measure.

Define a differential operator L as follows:

$$LV(x,t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x(t),x(t-d(t)),t) + \frac{1}{2} Tr\left\{ g^T \frac{\partial^2 V}{\partial x^2} g \right\}, \tag{2}$$

where $V(x,t) \in C^{2,1}$.