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Rich dynamics in a spatial predator–prey model with delay

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ABSTRACT

In this paper, we study the spatiotemporal dynamics of a diffusive Holling–Tanner predator–prey model with discrete time delay. Via analytically and numerically analysis, we unveil six types of patterns with and without time delay. Among them, of particular novel is the observation of linear pattern (consisting of a series of parallel lines), whose formation is closely related with the temporal Hopf bifurcation threshold. Moreover, we also find that larger time delay or diffusion of predator may induce the extinction of both prey and predator. Theoretical analysis and numerical simulations validate the well-known conclusion: diffusion is usually beneficial for stabilizing pattern formation, yet discrete time delay plays a destabilizing role in the generation of pattern.

1. Introduction

Pattern formation in reaction–diffusion system is an interesting and challenging problem in modern ecology, biology, chemistry, and other science fields. The earliest study of this issue could come back to the pioneering work of Turing [1], where the reaction–diffusion theory was proposed to inspect the range of spatial patterns observed in the developing embryo. Immediately after this seminal idea, the reaction–diffusion model and its various underlying applications have been intensively explored to explain pattern formation. Typical examples include patterns in fish skin, mammalian coat markings, phyllotaxis, predator–prey systems, terrestrial vegetation, plankton and intertidal communities [2–11]. More specifically, take the spatiotemporal model of predator–prey interactions as an instance. Patterns are mainly employed to understand the effect of individual mobility on the stable and oscillatory states of species survival. Along this way, there has been a great deal of accumulated achievements to enrich the spatial pattern formation within predator–prey model [12–17].

In population dynamics, the function response refers to the change in predator's rate of prey consumption with change in prey density. The most important and useful functional response is the Holling type II function. In line with this functional response, another well-known proposal is the well-known Holling–Tanner predator–prey temporal (HTT) model [18], which can well feature the realistic interactions between species in ecological systems [19,20]. Up to now, it has attracted considerable attention with mathematical analysis of dynamics, such as limit cycles [21], global stability [22] and bifurcation condition [23]. Moreover, it is clear that the spatial distribution of species is usually inhomogeneous. To capture this fact, Holling–Tanner predator–prey spatiotemporal (HTS) model has been extensively investigated as well. For example, Wang

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et al. studied the spatial patterns of this model with nonlinear cross diffusion and showed the asymptotic behavior of positive solutions [24]. With respect to the impact of HTS model on pattern formation, we can also refer to literature [25–27].

Besides the aforementioned progress, the study of reaction–diffusion system coupled with time delay, which in general exhibits more complex dynamics behavior, attracts much interest as well [28–31]. This is because delayed time will destabilize the evolution and bring more fluctuation into system. However, different from time delay, diffusion usually play a stabilizing role in the spatiotemporal system. In this sense, it seems very important to derive which factor will be responsible for the inhomogeneous spatial patterns. In recent researches [32,33], the authors obtained the Turing bifurcation threshold by considering the discrete time delay as a bifurcation parameter, which yet was just valid for small delay. While for the delayed spatiotemporal model, it would be difficult to derive the Turing bifurcation threshold. Moreover, Piotrowska and Banerjee derived the critical threshold for the spatiotemporal Hopf bifurcation by considering the delay as bifurcation parameter and further analyzed the influence of time delay on spatiotemporal pattern formation [34,35].

As what we descried, various types of spatial patterns can be driven by Turing instability (namely Turing pattern). However, some other factors may also play a potential role in pattern formation. In the present work, we are interested to explore how the discrete time delay as well as diffusion of the predator and prey affect the pattern formation. Different from the exiting work [35], which obtains Hopf bifurcation threshold by considering the delay as the parameter, we select more parameters and acquire more kinds of regular patters. Predator and prey in delayed HTS (DHTS) model become extinct for large delay, otherwise show chaos behavior. In addition, it is uncovered that the coexistence or extinction of predator and prey is more sensitive to the Hopf threshold of Delayed HTT (DHTT) model than the corresponding threshold in [35]. In the remainder of this paper, we will first describe the dynamics of HTT/DHTT model and HTS/DHTS model; subsequently, we will present various spatial patterns and investigate how the delay and diffusion affects pattern formation; finally we will summarize our conclusions.

2. The dynamics of temporal model

In this section, we consider the dynamics of the Holling–Tanner predator–prey temporal (HTT) model with discrete time delay. But before this, we need to mention some brief results about HTT model without time delay. For more discussion details, they are also available in [22].

The HTT model takes the following form:

$$\begin{cases} \frac{dU}{dT} = rU\left(1-\frac{U}{K}\right) - \frac{cUV}{U+m}, & T>0, \\ \frac{dV}{dT} = sV\left(1-\frac{hV}{U}\right), & T>0, \end{cases} \tag{1}$$

subjecting to positive initial condition $U(0),\ V(0) > 0$. Here $U(T)$ and $V(T)$ stand for the density of prey and predator. The meaning of positive constants $r,\ K,\ c,\ m,\ s,\ h$ are given in Table 1. For simplicity, we can also write the model (1) into non-dimensional form. Assuming

$$u = \frac{U}{K}, \quad v = \frac{cV}{Kr}, \quad t = rT, \quad \alpha = \frac{m}{K}, \quad \beta = \frac{hs}{c}, \quad \gamma = \frac{s}{r},$$

then Eq. (1) becomes

$$\begin{cases} \frac{du}{dt} = u(1-u) - \frac{uv}{u+\alpha}, \\ \frac{dv}{dt} = v\left(\gamma - \frac{\beta v}{u}\right). \end{cases} \tag{2}$$

Simple computation reveals that, apart from the saddle point $E_0(1,0)$, the model (2) possess a unique positive equilibrium point $E^*(u^*, v^*)$, where

$$u^* = \frac{(\beta-\alpha\beta-\gamma) + \sqrt{(\beta-\alpha\beta-\gamma)^2 + 4\alpha\beta^2}}{2\beta}, \quad v^* = \frac{\gamma}{\beta}u^*. \tag{3}$$

It is not difficult to find that $0 < u^* < 1,\ 0 < v^* < \frac{\gamma}{\beta}$. The Jacobian matrix evaluated for the system (2) at E^* is given by

Table 1
The meaning of some constants in (1).

r	Prey intrinsic growth rate
K	Carrying capacity
c	Capturing rate
m	Half capturing saturation constant
s	Predator intrinsic growth rate
h	Conversion rate of prey into predators biomass