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On a nonautonomous competitive system subject to stochastic and impulsive perturbations

Ronghua Tan [a,b], Zhijun Liu [a,b,*], Shengliang Guo [b], Huili Xiang [b]

[a] Key Laboratory of Biologic Resources Protection and Utilization of Hubei Province, Hubei University for Nationalities, Enshi, Hubei 445000, PR China
[b] Department of Mathematics, Hubei University for Nationalities, Enshi, Hubei 445000, PR China

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ABSTRACT

This paper presents a nonautonomous impulsive stochastic differential model of competition between species which contains the classic two-species competitive Lotka–Volterra model as a special case. With the theory of impulsive stochastic differential equations, a good understanding of stochastic permanence and extinction of system is obtained. Two specific numerical examples are presented to support the analytical results. Conclusions are also given.

1. Introduction

It is well known that Lotka [1] and Volterra [2] proposed independently a widely used model of interspecies competition

$$\begin{cases} x_1'(t) = x_1(t)[r_1 - a_{11}x_1(t) - a_{12}x_2(t)], \\ x_2'(t) = x_2(t)[r_2 - a_{21}x_1(t) - a_{22}x_2(t)], \end{cases} \tag{1.1}$$

where x_i is the number of individuals of species i at time t; r_i is the intrinsic growth rate of species i; a_{ii} and a_{ij} are, respectively, the coefficient of intra-specific competition and inter-specific competition. The above Lotka–Volterra model is subject to some criticisms since it assumes that competitive interactions, both intra-specific and inter-specific, are linear. Later, Ayala et al. [3] proposed a possible theoretical model of competition between species by adding a nonlinear term of self-interaction $(-b_i x_i^2)$ and obtained

$$\begin{cases} x_1'(t) = x_1(t)\left[r_1 - a_{11}x_1(t) - a_{12}x_2(t) - b_1x_1^2(t)\right], \\ x_2'(t) = x_2(t)\left[r_2 - a_{21}x_1(t) - a_{22}x_2(t) - b_2x_2^2(t)\right]. \end{cases} \tag{1.2}$$

They pointed out that model (1.2) is very good by experimental tests. For the relevant ecology of model (1.2) we refer the readers to Ref. [3]. To consider the influences of impulsive and stochastic factors on model (1.2) in a fluctuating environment, it is reasonable to study a corresponding ISDEs (impulsive stochastic differential equations) which may make for a suitable model as follows

* Corresponding author at: Department of Mathematics, Hubei University for Nationalities, Enshi, Hubei 445000, PR China.
E-mail address: zhijun_liu47@hotmail.com (Z. Liu).

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$$\begin{cases} \left.\begin{aligned} dx_1(t) &= x_1(t)\left[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - b_1(t)x_1^2(t)\right]dt \\ &\quad + \sigma_1(t)x_1(t)dw_1(t) \\ dx_2(t) &= x_2(t)\left[r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - b_2(t)x_2^2(t)\right]dt \\ &\quad + \sigma_2(t)x_2(t)dw_2(t) \end{aligned}\right\} t \neq \tau_k, \\ \left.\begin{aligned} x_1(\tau_k^+) &= (1+\lambda_{1k})x_1(\tau_k) \\ x_2(\tau_k^+) &= (1+\lambda_{2k})x_2(\tau_k) \end{aligned}\right\} t = \tau_k, \quad k \in \mathbb{N}. \end{cases} \tag{1.3}$$

Here the coefficients $r_i(t), a_{ij}(t), b_i(t)$ and $\sigma_i(t)$ are positive continuous bounded functions on $R_+ = [0, +\infty)$. $\mathbb{N}$ is the set of positive integers. The impulsive perturbations satisfy $\lambda_{ik} > -1$, in particular, $\lambda_{ik} > 0$ stand for stocking while $\lambda_{ik} < 0$ mean harvesting (see Refs. [4–8]). Also, we assume that the environmental noise mainly affects the intrinsic growth rate of species i with

$$r_i(t) \to r_i(t) + \sigma_i(t)\dot{w}_i(t), \quad i = 1,2, \tag{1.4}$$

where $\dot{w}_i(t)$ are white noises and $\sigma_i^2(t)$ represent the intensities of the noises. $w_i(t)$ are the independent standard Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geqslant 0}, \mathbb{P})$ with a filtration $\{\mathcal{F}\}_{t\geqslant 0}$ satisfying the usual conditions.

Throughout this paper, it is assumed that

(S_1) The impulsive points τ_k satisfy $0 < \tau_1 < \tau_2 < \cdots, \lim_{k\to+\infty}\tau_k = +\infty, k \in \mathbb{N}$.

(S_2) There exist positive constants m_i and M_i such that

$$m_i \leqslant \prod_{0<\tau_k<t} (1+\lambda_{ik}) \leqslant M_i, \quad i = 1,2 \tag{1.5}$$

and a product equals unity if the number of factors is zero.

This is a continuation of the work of Tan et al. [9]. In [9], we have established an almost periodic version of model (1.3) without stochastic perturbations and discussed the uniformly asymptotic stability of almost periodic solutions. In this contribution, we further consider the factor of stochastic perturbations and focus on the asymptotic behaviors of system (1.3). Our motivation comes from the work of Liu and Wang [10]. The rest structure of this paper is as follows. In Section 2, some preliminaries are introduced. In Section 3, we first investigate the existence and uniqueness of positive solution of system (1.3) and later derive sufficient conditions for stochastic permanence and extinction. In Section 4, we present two specific numerical examples to verify the feasibility of our analytical results. Conclusions are given in the final section.

2. Preliminaries

Denote R_+^2 by

$$R_+^2 = \{x \in R^2 : x_1 > 0, \quad x_2 > 0\}.$$

If $x \in R^2$, its norm is $|x| = \sqrt{x_1^2 + x_2^2}$. For a continuous and bounded function on $f(t)$, we use the notations

$$f^l = \inf_{t\in R_+} f(t), \quad f^u = \sup_{t\in R+} f(t).$$

Also, for any constant sequences $\{A_{ij}\}$ and $\{B_i\}, 1 \leqslant i,j \leqslant 2$, we define

$$\hat{A} = \min_{1\leqslant i,j\leqslant 2} A_{ij}, \quad \check{A} = \max_{1\leqslant i,j\leqslant 2} A_{ij}, \quad \hat{B} = \min_{1\leqslant i\leqslant 2} B_i, \quad \check{B} = \max_{1\leqslant i\leqslant 2} B_i.$$

Definition 2.1 [11]. Consider the following ISDEs

$$\begin{cases} dX(t) = F(t, X(t))dt + G(t, X(t))dW(t), \quad t \neq \tau_k, \\ X(\tau_k^+) - X(\tau_k) = B_k X(\tau_k), \quad t = \tau_k, \quad k \in \mathbb{N}, \end{cases} \tag{2.1}$$

with initial condition $X(0)$. If a stochastic process $X(t) = (X_1(t), \ldots, X_n(t)), t \in R_+$, satisfies the following

(1) $X(t)$ is $\mathcal{F}_t$-adapted and is continuous on $(0, \tau_1)$ and each interval $(\tau_k, \tau_{k+1}), k \in N$; $F(t, X(t)) \in L^1(R_+; R^n)$, $G(t, X(t)) \in L^2(R_+; R^n)$, where $L^k(R_+; R^n)$ is all R^n valued measurable $\mathcal{F}_t$-adapted processes $\phi(t)$ satisfying $\int_0^T |\phi(t)|^k dt < +\infty$ a.s. for every $T > 0$.