Applied Mathematics and Computation 256 (2015) 715–723

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Legendre wavelets operational method for the numerical solutions of nonlinear Volterra integro-differential equations system

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ARTICLE INFO

Keywords:
Legendre wavelets
Integro-differential equations
System of nonlinear Volterra integral equations
Legendre wavelet method

ABSTRACT

In this paper, Legendre wavelet method is developed to approximate the solutions of system of nonlinear Volterra integro-differential equations. The properties of Legendre wavelets are first presented. The properties of Legendre wavelets are used to reduce the system of integral equations to a system of algebraic equations which can be solved numerically by Newton's method. Also, the results obtained by present method have been compared with that of by B-spline wavelet method. Illustrative examples have been discussed to demonstrate the validity and applicability of the present method.

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1. Introduction

Wavelets theory is a relatively new and an emerging area in the field of applied science and engineering. It has been applied in a wide range of engineering disciplines; particularly, wavelets are very successfully used in signal analysis for waveform representation and segmentation, time–frequency analysis and fast algorithms for easy implementation [1]. Wavelets permit the accurate representation of a variety of functions and operators. Moreover wavelets establish a connection with fast numerical algorithms [2]. Many researchers started using various wavelets for analyzing problems of high computational complexity. It is proved that wavelets are powerful tools to explore new direction in solving differential equations and integral equations.

Integral equation has been one of the essential tools for various areas of applied mathematics. Integral equations occur naturally in many fields of science and engineering [3]. Mathematical modeling of real-life problems usually results in functional equations, e.g. partial differential equations, integral and integro-differential equations, stochastic equations and others. Many mathematical formulations of physical phenomena contain integro-differential equations; these equations arise in fluid dynamics, biological models and chemical kinetics. Integro-differential equations arises in many physical processes, such as glass-forming process [4], nano-hydrodynamics [5], drop wise condensation [6], wind ripple in the desert [7] and biological model [8].

In the past several decades, many effective methods for obtaining approximation or numerical solutions of linear and nonlinear integro-differential equations have been presented. There are various numerical and analytical methods to solve such problems.

In the past, Legendre wavelet method (LWM) has been applied to solve the integral equations and integro-differential equations of different forms. Recently, Mohamed and Torky [9] have solved system of linear Fredholm and Volterra integral

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http://dx.doi.org/10.1016/j.amc.2015.01.063
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equation by applying Legendre wavelet method. In [10], the learned researchers Venkatesh et al. have applied Legendre wavelet method to solve class of nonlinear integro-differential equations. Also the Legendre wavelet method has been applied to solve nonlinear Volterra–Fredholm integral equations by Yousefi and Razzaghi in [11], and system of Fredholm integral equations by Biazer and Ebrahimi in [12]. Biazer et al. in [13] have solved system of nonlinear Volterra integro-differential equations by using Homotopy perturbation method. In [14], system of linear Volterra integro-differential equations has been solved by using Variational iteration method by Nadjafi and Tamamgar. Maleknejad et al. [15] have solved system of high order linear Volterra Integro-differential equations by applying Bernstein operational matrix. Numerical methods for solving Fredholm integral equations have been presented by learned researchers Saha Ray and Sahu in [16]. Linear semi-orthogonal B-spline wavelets have been applied to solve integral equations and systems in [16–18].

In this paper, we consider the system of nonlinear Volterra integro-differential equations of the following form

$$y_i^{(p)}(x) = G_i(x, \mathbf{Y}(x)) + \sum_{j=1}^{l} \int_0^x k_{i,j}(x,t) F_{i,j}(t, \mathbf{Y}(t))dt, \quad i = 1, 2, \ldots, l. \tag{1.1}$$

with initial conditions $y_i^{(s)}(0) = \beta_{i,s}, \quad s = 0, 1, \ldots, p-1, \quad i = 1, 2, \ldots, l$, where

$$G_i(x, \mathbf{Y}(x)) = G_i\left(x, y_1(x), y_1^{(1)}(x), \ldots, y_1^{(p)}(x), \ldots, y_l(x), y_l^{(1)}(x), \ldots, y_l^{(p)}(x)\right),$$
$$F_{i,j}(t, \mathbf{Y}(t)) = F_{i,j}\left(t, y_1(t), y_1^{(1)}(t), \ldots, y_1^{(p)}(t), \ldots, y_l(t), y_l^{(1)}(t), \ldots, y_l^{(p)}(t)\right),$$

and $k_{i,j}(x,t)$ are the kernel functions for $i,j = 1, 2, \ldots, l$ and $y_i^{(p)}(x)$ is the pth order derivative of $y_i(x)$ for $i = 1, 2, \ldots, l$.

In this present paper, we apply Legendre wavelet method (LWM) to solve the system of nonlinear Volterra integro-differential equations. The Legendre wavelet method converts the system of integro-differential equations to a system of algebraic equations and that algebraic equations system again can be solved by any of the usual numerical methods. The results obtained by present method have been compared with the results obtained by B-spline wavelet method (BWM). In aid to compare, the same solution method has been implemented to both the methods.

2. Properties of Legendre wavelets

Wavelets constitute a family of functions constructed from dilation and translation of a single function called mother wavelet. When the dilation parameter a and the translation parameter b vary continuously, we have the following family of continuous wavelets as

$$\Psi_{a,b}(x) = |a|^{-\frac{1}{2}} \Psi\left(\frac{x-b}{a}\right), \quad a, b \in \mathbb{R}, \quad a \neq 0 \tag{2.1}$$

If we restrict the parameters a and b to discrete values as $a = a_0^{-k}, b = nb_0 a_0^{-k}, a_0 > 1, b_0 > 0$ and n, and k are positive integers, from Eq. (2.1) we have the following family of discrete wavelets:

$$\psi_{k,n}(x) = |a_0|^{-\frac{k}{2}} \psi(a_0^k x - nb_0),$$

where $\psi_{k,n}(x)$ form a wavelet basis for $L^2(\mathbb{R})$. In particular, when $a_0 = 2$ and $b_0 = 1$, then $\psi_{k,n}(x)$ form an orthonormal basis.

Legendre wavelets $\psi_{n,m}(x) = \psi(k, \hat{n}, m, x)$ have four arguments. $\hat{n} = 2n - 1,\ n = 1, 2, \ldots, 2^{k-1},\ k \in \mathbb{Z}^+, m$ is the order of Legendre polynomials and x is normalized time. They are defined on $[0, 1)$ as

$$\psi_{n,m}(x) = \psi(k, \hat{n}, m, x) = \begin{cases} \sqrt{m + \frac{1}{2}}\, 2^{\frac{k}{2}} P_m(2^k x - \hat{n}), & \frac{\hat{n}-1}{2^k} \leqslant x < \frac{\hat{n}+1}{2^k} \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

where $m = 0, 1, \ldots, M-1$ and $n = 1, 2, \ldots, 2^{k-1}$. The coefficient $\sqrt{m + \frac{1}{2}}$ is for orthonormality, the dilation parameter is $a = 2^{-k}$ and translation parameter is $b = \hat{n} 2^{-k}$.

Here $P_m(x)$ is Legendre polynomials of order m, which are orthogonal with respect to weight function $w(x) = 1$ on the interval $[-1, 1]$. This can be determined from the following recurrence formulae

$$P_0(x) = 1,$$
$$P_1(x) = x,$$
$$P_{m+1}(x) = \left(\frac{2m+1}{m+1}\right) x P_m(x) - \left(\frac{m}{m+1}\right) P_{m-1}(x), \quad m = 1, 2, 3, \ldots$$

3. Function approximation by Legendre wavelets

A function $f(x)$ defined over $[0, 1)$ can be expressed by the Legendre wavelets as

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