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Resolving-power dominating sets

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ABSTRACT

For a graph $G(V,E)$ that models a facility or a multi-processor network, detection devices can be placed at vertices so as to identify the location of an intruder such as a thief or fire or saboteur or a faulty processor. Resolving-power dominating sets are of interest in electric networks when the latter helps in the detection of an intruder/fault at a vertex. We define a set $S \subseteq V$ to be a resolving-power dominating set of G if it is resolving as well as a power-dominating set. The minimum cardinality of S is called resolving-power domination number. In this paper, we show that the problem is *NP*-complete for arbitrary graphs and that it remains *NP*-complete even when restricted to bipartite graphs. We provide lower bounds for the resolving-power domination number for trees and identify classes of trees that attain the lower bound. We also solve the problem for complete binary trees.

1. Introduction

A *dominating* set of a graph $G(V,E)$ is a set S of vertices such that every vertex (node) in $V \setminus S$ has at least one neighbor in S. The problem of finding a dominating set of minimum cardinality is an important problem that has been extensively studied. The minimum cardinality of a dominating set of G is its *domination number*, denoted by $\gamma(G)$ [5]. In a connected graph G, the distance $d(u,v)$ between two vertices $u, v \in V$ is the length of a shortest path between them. Let $W = \{w_1, w_2 \dots w_k\}$ be an ordered set of vertices of G and let v be a vertex of G. The representation $r(v|W)$ of v with respect to W is the k-tuple $(d(v,w_1), d(v,w_2) \dots d(v,w_k))$. If distinct vertices of G have distinct representations with respect to W, then W is called a *locating/resolving* set for G [10,2]. A resolving set of minimum cardinality is called a *metric basis* for G and this cardinality is the *metric dimension* of G, denoted by $dim(G)$ [3]. A set $S \subseteq V$ is called a *metric-locating-dominating* set if the set S is both a locating as well as a dominating set. The minimum cardinality of such a set is called *metric-locating-domination number* denoted by $\eta(G)$ [6]. Our focus is on a variation called the resolving-power dominating set (RPDS) problem. For a vertex v of G, let $N(v)$ and $N[v]$ denote the open and closed neighborhoods of v respectively. For a set S, let $N(S) = \cup_{v \in S} N(v) \setminus S$ and $N[S] = N(S) \cup S$ denote the open and close neighborhoods of S respectively. For vertices $x, y \in V$, let the notation $x \sim y$ mean that x is adjacent to y.

The power domination problem arose in the context of monitoring electric power networks. A power network contains a set of nodes and a set of edges connecting the nodes. It also contains a set of generators, which supply power, and a set of loads, where the power is directed to. In order to monitor a power network we need to measure all the state variables of the network by placing measurement devices. A Phase Measurement Unit (PMU) is a measurement device placed on a node that

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has the ability to measure the voltage of the node and current phase of the edges connected to the node. The goal is to install the minimum number of PMUs such that the whole system is monitored. This problem has been formulated as a graph domination problem by Haynes et al. in [4]. However, this type of domination is different from the standard domination type problem, since the domination rules can be iterated. The propagation rules are derived from the Ohm's and Kirchoff's laws for an electric circuit.

Let the graph $G(V, E)$ represent an electric power system, where a vertex represents an electrical component such as a PMU and an edge represents a transmission line joining two electrical nodes. A PMU measures the state variable for the vertex at which it is placed as well as its incident edges and their end vertices (these vertices and edges are said to be observed).

The other observation rules are as follows:

1. Any vertex that is incident to an observed edge is observed.
2. Any edge joining two observed vertices is observed.
3. If a vertex is incident to a total of $k > 1$ edges and if $k - 1$ of these edges are observed, then all k of these edges are observed.

A set $S \subseteq V$ is called a *power dominating set* (PDS) of G if every vertex and edge of G are observed. The *power domination number* $\gamma_p(G)$ is the minimum cardinality of a PDS of G. A PDS of G with the minimum cardinality is called a $\gamma_p(G)$-set. Since any dominating set is a power dominating set, $1 \leqslant \gamma_p(G) \leqslant \gamma(G)$ for all graphs G [4].

A set $S \subseteq V$ is called a *resolving-power dominating* set if the set S is both a resolving as well as a power dominating set. The minimum cardinality of such a set is called *resolving-power domination number* denoted by $\eta_p(G)$. We shall illustrate with an example.

In the example in Fig. 1, $dim(G) = 2, \gamma_p(G) = 2, \eta(G) = 4$ and $\eta_p(G) = 3$.

The following results are straight forward.

Theorem 1.1. *$max\{\gamma_p(G), dim(G)\} \leqslant \eta_p(G) \leqslant \gamma_p(G) + dim(G)$.*

Proof. Suppose $\eta_p(G) < max\{\gamma_p(G), dim(G)\}$. Let $S \subseteq V$ be a resolving-power dominating set of cardinality $\eta_p(G)$. But this is a contradiction to the definition of $\gamma_p(G)$ and $dim(G)$. Consider a set S which comprises of all vertices in metric basis B and all vertices in the power dominating set D. It is easy to see that S is a resolving-power dominating set. Thus, $\eta_p(G) \leqslant \gamma_p(G) + dim(G)$

Theorem 1.2. *$\eta_p(G) = 1$ if and only if G is a path.*

Proof. If G is a path, then by Theorem 1.1, $\eta_p(G) \geqslant 1$. For the reverse inequality, it is easy to see that any pendant vertex of G is a RPDS.

Suppose now $\eta_p(G) = 1$ and G is not a path. Then G is either a cycle or has a vertex v such that $d(v) \geqslant 3$. In both cases, by Theorem 1.1, $\eta_p(G) \geqslant 2$, a contradiction to the assumption.

Theorem 1.3. *$\eta_p(G) = n - 1$ if and only if G is the complete graph K_n.*

Proof. If G is K_n, then by Theorem 1.1, $\eta_p(G) \geqslant n - 1$. This is due to the fact that $dim(K_n) = n - 1, \gamma_p(K_n) = 1$ and choosing any set of $n - 1$ vertices of K_n would be a resolving-power dominating set. Conversely assume that $\eta_p(G) = n - 1$ and

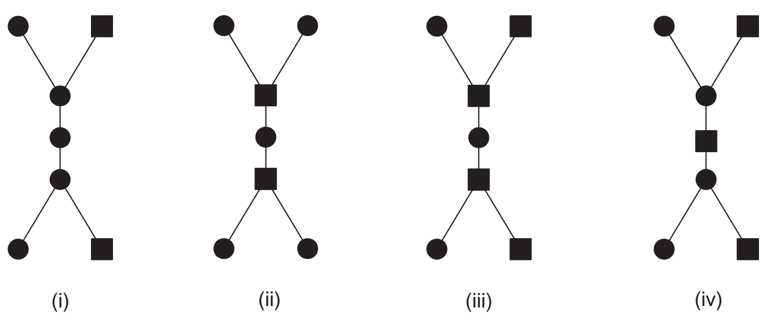

Fig. 1. Square bullets depict (i) metric basis (ii) minimum power dominating set (iii) minimum metric-locating-dominating set and (iv) minimum resolving-power-dominating set of G.