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Impact of density and interconnectedness of influential players on social welfare



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ABSTRACT

We show that in defection prone environments influential players must be rare and weakly interconnected to optimally promote cooperation in the prisoner's dilemma game. Conversely, low temptations to defect warrant a high level of social welfare even if influential players are common, yet still demand the latter be weakly interconnected for cooperation to thrive.

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1. Introduction

Cooperation between competitive players or firms is a perplexing puzzle faced by scientists across fields of research as different as economics and anthropology [10]. The core of the problem lies in the fact that cooperation implies working for the common good of a society on the expense of individual prosperity, which is in contradiction with the premise of Darwinian evolution and results in a social dilemma. The most commonly adopted theoretical framework for addressing the issue is the evolutionary game theory, and the prisoner's dilemma game in particular [1], which succinctly captures the unadorned scenario of social downfall due to the egoistic and heedless urge of individuals to outperform competitors by temporarily harvesting the highest possible profit. In particular, the game originally consists of two players or firms who have to decide simultaneously whether they want to cooperate or defect as they venture into a joint enterprise. The dilemma is given by the fact that although mutual cooperation yields the highest collective payoff, a defector will do better if the opponent cooperates. In the long run this fact inflicts mutual defection that ultimately results in an irreversible economic decline and social poverty, as reviewed by Crawford [4].

The breakthrough discovery promoting cooperation arguably came in the form of spatial games [11,16], where the participating players no longer abide to the principles of well-mixed dynamics, but instead, cooperators are able to survive via clustering that protects the inner individuals against the exploitation by invading defectors. This approach has recently been extended further by considering structured populations where interactions amongst individuals are defined by complex networks [17], and also, by the introduction of asymmetry in influence [18] as well as payoff uncertainties [12,13], thus revealing additional mechanisms promoting mutually beneficial alliances not just in economy (e.g., [9] or [3], but also in ecology and biology in general.

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Presently, we study the impact of two crucial factors on the evolution of cooperation within the prisoner's dilemma game; namely the density and interconnectedness of influential players. The basic premise of our study is that influence is seldom equally distributed amongst members of human societies. Throughout history, many phrases and titles have been invented to distinguish influential individuals from those having little impact, and more often than not "being influential" is reserved for few selected individuals only. To take this fact into account we introduce a fraction ρ , of influential players on the grid, which are much more likely to enforce their strategy on the opponent than the rest. In addition, amongst all the possible connections existing between the influential players, we introduce a fraction κ that actually exist, and study how different values of ρ and κ affect the evolution of cooperation by different temptations to defect b. First, we show that the introduction of influential players and shortcut connections amongst them in general facilitates cooperation, which suggests that the role of leaders in societies might have emerged spontaneously in order to warrant optimal conditions for cooperative alliances and with it related social welfare. Yet more precisely, we reveal that in defection prone environments governed by large b influential players must be rare ($\rho << 1$) and weakly connected ($\kappa << 1$) to sustain cooperation, whereas on the other hand, by small b a high level of social welfare is warranted even if ρ is substantial, but still κ must remain small for cooperation to flourish best. Importantly, all results are obtained by full anonymity of players and incognito actions (for related literature see e.g., [5] or [15], as well as without the aid of additional strategies (see e.g., [14]. Finally we discuss that, although large differences in status may evoke dissatisfaction and rebellion amongst the deprived, our study suggests that such social states might have appeared spontaneously from an initially nonpreferential setup and are in fact optimal for the evolution of cooperation.

2. Mathematical model

For the purpose of this study, we used the prisoner's dilemma game having temptation to defect T = b, reward R = 1, and both punishment P as well as the suckers payoff S equaling S0, whereby S1 < S2 ensures a proper payoff ranking. The game is staged on a regular S2 k square grid with nearest neighbor interactions and periodic boundary conditions, whereon initially each agent on site S3 is designated either as a cooperator (strategy S3 c S4 or defector (S3 c S4 S5) with equal probability. Forward iteration is performed in accordance with the Monte Carlo simulation procedure comprising the following elementary steps. First, a randomly selected agent S4 acquires its payoff S5 by playing the game with all its neighbors (here neighbors are assumed to be all players that are directly connected to agent S5. Next, one randomly chosen neighbor, denoted by S5, acquires its payoff S7 by playing the game with all its neighbors. Last, agent S5 tries to enforce its strategy S6 on player S7 in accordance with the probability

$$W(s_y \to s_x) = w_x \frac{1}{1 + \exp[(P_y - P_x)/K]},$$
 (1)

where K=0.5 denotes the level of uncertainty by strategy adoptions due to imperfect information (see e.g., [15] and w_x determines the influence of agent x. The parameter w_x is assigned to each agent at the beginning of the game and remains fixed during the evolutionary process. In particular, amongst all $N=L^2$ players, and irrespective of their initial strategies, a fraction ρ is chosen randomly and designated as having $w_x=1$, whereas the remaining $N(1-\rho)$ are assigned the influence $w_x=0.01$. In accordance with Eq. (1), players characterized by $w_x=1$ are 100 times more likely to enforce their strategy on the opponent than those having $w_x=0.01$, and are thus termed as the influential ones. Furthermore, we introduce a probability κ that determines the fraction of all possible $(\rho N-1)\rho N/2$ connections linking the influential individuals that actually exist. If $\kappa=1$ all possible pairs of influential players are connected with one another as depicted in Fig. 1(a), whereas if $0<\kappa<1$ the corresponding fraction of some of these links is randomly removed as shown in Fig. 1(b). Finally, the network characterizing the interconnectedness of influential players is merged with the underlying nearest-neighbor grid to form the final interaction structure for the considered evolutionary prisoner's dilemma game.

Results obtained via Monte Carlo simulations presented below were obtained on populations comprising 400×400 individuals, whereby the stationary fraction of cooperators f_C was determined within 10^6 full Monte Carlo steps after sufficiently long transients were discarded. In what follows, we will focus on different values of density ρ and interconnectedness κ of influential players to determine their impact on cooperation and social welfare by different values of the temptation to defect b.

3. Results and discussion

We start by presenting results obtained with the classical version of the prisoner's dilemma game when all players have the same probability of enforcing their strategy on the opponents and there are no additional shortcut links defining the interaction structure amongst them. In accordance with the above description of the game, this setup is obtained by setting $\kappa = 0$ and $\rho = 0$. Results presented in Fig. 1 (circles) evidence that under such conditions cooperators vanish already by b = 1.065. In sharp contrast, by setting $\kappa = 0.0012$ and $\rho = 0.15$, cooperators persist virtually throughout the whole range of $b \in (1, 2]$ applicable for the prisoner's dilemma game, as depicted by squares in Fig. 1. Noteworthy, by introducing this fairly small fraction of weakly linked influential players, the cooperators are able to dominate the game completely by $b \approx 1$ and outnumber defectors up to b = 1.15. Clearly, the presently studied additions to the game in form of positive κ

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