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Travelling wave solutions to some time–space nonlinear evolution equations

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ABSTRACT

In this article, the modified direct algebraic method has been extended to celebrate the new complex travelling wave solutions of nonlinear evolution equations of fractional order. For finding solutions, the fractional complex transformation has been implemented to convert nonlinear partial fractional differential equations into nonlinear ordinary differential equations. After this, the modified direct algebraic method has been applied successfully, to construct the complex travelling wave solutions. As far as concerned about the applications, the nonlinear time–space fractional KdV, foam drainage and coupled burgers' equations of fractional order have been considered.

1. Introduction

The nonlinear phenomena has not only shown a variety of applications in electromagnetic, acoustics, electrochemistry, cosmology and fluid mechanics, but has also shown many fundamental physical phenomena in nature that cannot be described without nonlinear evolution equations [1–4].

Knowing the importance of nonlinear evolution equations, a lot of techniques already exist to find solutions in the field of mathematical physics. In recent years, many new powerful and interesting techniques have been developed to handle the nonlinear equations. For example, adomian decomposition method [5] and generalized differential transform method [6] have been used to find the numerical solutions for fractional order differential equations. (G'/G)-expansion method [7] can also construct the travelling wave solutions of nonlinear evolution equations. This method has been further extended [8,9] to seek the solutions for nonlinear equations of fractional order. Jacobi elliptic function expansion method [10], tanh-function method [11], homotopy perturbation method [12] *etc.* were also developed for solving the nonlinear evolution equations. For more references see also [13–16].

In this article, fractional complex transformation [17] has been implemented to convert nonlinear partial differential equations into nonlinear ordinary differential equations, using Jumarie's modified Riemann–Liouville derivative [18]. The modified direct algebraic method [19,20] can be applied to find the exact travelling wave solutions from ODE. For this, the following applications have been considered to find new complex solutions. Firstly, time–space fractional derivative nonlinear KdV equation, in the following form [8], has been considered:

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$$\frac{\partial^{\alpha}u}{\partial t^{\alpha}}+au\frac{\partial^{\beta}u}{\partial x^{\beta}}+\frac{\partial^{3\beta}u}{\partial x^{3\beta}}=0,\quad t>0,\ 0<\alpha,\ \beta\leqslant 1. \tag{1.1}$$

The time–space fractional derivative foam drainage equation [9] can be considered in the following form:

$$\frac{\partial^{\alpha}u}{\partial t^{\alpha}}=\frac{1}{2}u\frac{\partial^{2\beta}u}{\partial x^{2\beta}}+2u^{2}\frac{\partial^{\beta}u}{\partial x^{\beta}}+\left(\frac{\partial^{\beta}u}{\partial x^{\beta}}\right)^{2},\quad t>0,\ 0<\alpha,\ \beta\leqslant 1, \tag{1.2}$$

and in view of [8], the time–space fractional coupled burgers' equation has the following form:

$$\frac{\partial^{\alpha}u}{\partial t^{\alpha}}=\frac{\partial^{2\beta}u}{\partial x^{2\beta}}-2u\frac{\partial^{\beta}u}{\partial x^{\beta}}-a\frac{\partial^{\beta}(uv)}{\partial x^{\beta}}$$

$$\frac{\partial^{\alpha}v}{\partial t^{\alpha}}=\frac{\partial^{2\beta}v}{\partial x^{2\beta}}-2v\frac{\partial^{\beta}v}{\partial x^{\beta}}-b\frac{\partial^{\beta}(uv)}{\partial x^{\beta}},\quad t>0,\ 0<\alpha,\ \beta\leqslant 1. \tag{1.3}$$

The rest of the article is organized as follows, in Section 2 the basic definitions and properties of the fractional calculus have been considered. In Section 3, the modified direct algebraic method has been proposed to find the new exact solutions for NPDEs of fractional order with the help of fractional complex transformation. Some applications, in Section 4, to construct the new exact solutions of time–space fractional KdV, foam drainage and coupled burgers' equations of fractional order have been considered. The conclusion is drawn in last Section 5.

2. Preliminaries and basic definitions

In this section, the extended method has been applied in the sense of Jumarie's modified Riemann–Liouville derivative of order α. For this, some basic definitions and properties of fractional calculus are being considered (for details see also [3]).

Definition 2.1. Jumarie's modified Riemann–Liouville derivative, of order α, can be defined as follows:

$$D_s^{\alpha}f(s)=\begin{cases}\frac{1}{\Gamma(1-\alpha)}\frac{d}{ds}\int_0^s(s-\xi)^{-\alpha}(f(\xi)-f(0))d\xi, & 0<\alpha<1,\\ \left(f^{(n)}(s)\right)^{\alpha-n}, & n\leqslant\alpha<n+1,\ n\geqslant 1.\end{cases}$$

Moreover, some properties for the modified Riemann–Liouville derivative have also been considered,

$$D_s^{\alpha}s^{r}=\frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)}s^{r-\alpha},$$
$$D_s^{\alpha}(f(s)g(s))=f(s)D_s^{\alpha}g(s)+g(s)D_s^{\alpha}t(s),$$
$$D_s^{\alpha}f[g(s)]=f'_g[g(s)]D_s^{\alpha}g(s)=D_s^{\alpha}f[g(s)](g'(t))^{\alpha}.$$

The above properties are very useful and help to convert fractional order differential equations into ordinary differential equations. In the following section, the extended modified direct algebraic method has been described to find the solutions.

3. The modified direct algebraic method

In this section, the modified direct algebraic method [19,20] has been discussed to obtain the solutions of nonlinear partial differential equations of fractional order.

For this, we consider the NPDEs of fractional order in the following way:

$$P\Big(u,D_t^{\alpha}u,D_s^{\beta}u,D_x^{\gamma}u,\ldots,D_t^{\alpha}D_t^{\alpha}u,D_t^{\alpha}D_s^{\beta}u,D_s^{\beta}D_s^{\beta}u,D_s^{\beta}D_x^{\gamma}u,\ldots\Big)=0,\quad for\ 0<\alpha,\beta,\gamma<1, \tag{3.1}$$

where u is an unknown function and P is a polynomial of u and its partial fractional derivatives along with the involvement of higher order derivatives and nonlinear terms.

To find the exact solutions, the method can be performed using the following steps.

Step 1: First, we convert the NPDEs of fractional order into nonlinear ordinary differential equations using fractional complex transformation introduced by Li and He [17]. For this, the travelling wave variable

$$u(t,x,y)=u(\xi),\xi=\frac{Kt^{\alpha}}{\Gamma(\alpha+1)}+\frac{Lx^{\beta}}{\Gamma(\beta+1)}+\frac{My^{\gamma}}{\Gamma(\gamma+1)} \tag{3.2}$$