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Three-dimensional Green's functions for a fluid and pyroelectric two-phase material

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ABSTRACT

The three-dimensional Green's functions for a steady-state point heat source in the interior of a fluid and pyroelectric two-phase material are presented in this paper. By virtue of the three-dimensional general solutions which are expressed in harmonic functions, six newly introduced harmonic functions with undetermined constants are constructed. Then, all the pyroelectric components in the fluid and pyroelectric two-phase material can be derived by substituting these harmonic functions into the corresponding general solutions. And the undetermined constants can be obtained by the interface compatibility conditions and the mechanical, electric and thermal equilibrium conditions. Numerical results are given graphically by contours.

1. Introduction

Green's functions play an important role in both applied and theoretical studies on the physics of solids. They are a basic building block of a lot of further works. For example, the Green's functions can be used to construct many analytical solutions of practical problems. They are also very important in the boundary element method as well as the study of cracks, defects and inclusions.

For purely elastic or thermoelastic solids, Green's functions had been well investigated and a great deal of works can be found in the literature. Lifshitz and Rozentsveig [1] and Lejcek [2] derived the Green's functions when a vertical point force acted on the surface of an anisotropic elastic half space by using the Fourier transform method. Elliott [3], Kroner [4] and Willis [5] studied the Green's functions by using the direct method. Sveklo [6] studied the Green's functions by using the complex method. Pan and Chou [7] proposed the Green's functions by using the potential function method. In this solution, the buried loads can be vertical or horizontal with respect to the boundary plane. When the thermal effects are considered, Melan and Parkus [8] presented the Green's functions for a point heat source on the surface of a semi-infinite body. Sharma [9] gave the Green's functions of transversely isotropic semi-infinite thermoelastic materials in integral form. Nowacki [10] presented the Green's functions for a point heat source in the interior of an infinite body. Yu [11] investigated the Green's function for two-phase isotropic thermoelastic materials. By virtue of the general solution of Chen et al. [12] and Hou et al. [13,14] constructed Green's function for a point heat source acted on infinite, semi-infinite and two-phase transversely isotropic thermoelastic materials. Besides, for anisotropic thermoelastic materials, Berger and Tewary [15] and Kattis et al. [16] obtained the two-dimensional Green's functions.

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For piezoelectric material with electromechanical coupling, Green's functions have also received much attention. For dislocation, crack, and inclusion problem in anisotropic piezoelectric solids, Deeg [17], Wang [18], Benveniste [19], Chen [20], and Chen and Lin [21] expressed the Green's function in the form of integral representation through the use of the transform techniques. Pan [22] derived the two-dimensional Green's functions of infinite, semi-infinite, and two-phase material by the complex function method. Gao and Fan [23] also gave the two-dimensional Green's functions of semi-infinite material. Pan and Tonon [24], Pan and Yuan [25] derived the Green's functions of infinite and two-phase material. With regards to the special case of transversely isotropic piezoelectric material, Sosa and Castro [26], Lee and Jiang [27], and Ding et al. [28–30] studied the two-dimensional Green's functions of infinite, semi-infinite, and two-phase material. Wang and Chen [31], Wang and Zheng [32] obtained the Green's function for point loads acted on the surface of semi-infinite piezoelectric material. Dunn [33] gave the Green' function of infinite piezoelectric material by using Radon transform, coordinate transformation, and evaluation of residues in sequence. Ding et al. [34,35], Dunn and Wienecke [36,37] obtained the Green's functions for the infinite, semi-infinite and two-phase piezoelectric material in terms of elementary functions, which were employed to study the inclusion problem [38].

When the thermal effects are considered, Qin [39–43] derived a series of two-dimensional Green's functions for the anisotropic pyroelectric material with cracks, holes and inclusions. Chen [44] derived a compact three-dimensional general solution for transversely isotropic pyroelectric materials. In this general solution, all components of the pyroelectric field are expressed by four harmonic functions. Based on this general solution, Chen et al. [45] derived the three-dimensional Green's function of transversely isotropic pyroelectric material with a penny-shaped crack. Xiong et al. [46] obtained the two-dimensional Green's functions for the semi-infinite pyroelectric material under a line heat source. Hou et al. [47–49] obtained the three-dimensional Green's functions for the infinite, semi-infinite and two-phase pyroelectric material under a point heat source.

In the electrical and mechanical engineering, it is not only needed to study the heat transfer between solids, but also to study the heat transfer between solid with fluid (including liquid and gas). For examples, the pyroelectric energy harvesting device is a kind of very promising energy harvesting device for the sustainable development [50–52]. This device is often located in the fluid to convert the heat energy into electric energy. In addition, pyroelectric sensors are widely used in engineering for the directional gas-flow measurement [53,54] and the thermal diffusivity measurements in fluids [55,56]. For these pyroelectric devices, the mutations of material properties between the interface of pyroelectric material and fluid will result in obvious interface effects. So that it is necessary to study the interface effects for the accurate design of these pyroelectric devices. The Green's functions for a point heat source in a fluid and pyroelectric two-phase material is the fundamental problem in this area. To the authors' knowledge, some literatures are concentrated to the heat transfer between solids [35,39,49], while the literature on heat transfer between solid with fluid is little reported.

Under this background, the three-dimensional Green's functions for a point heat source in a fluid and pyroelectric two-phase material are presented in this paper. For completeness, the governing equations and corresponding general solutions which are expressed with harmonic functions are introduced at first. Based on these general solutions, six new harmonic functions with undetermined constants are constructed for a point heat source applied in the interior of fluid and pyroelectric two-phase material. two cases including the free or fixed interface are considered, and all the corresponding pyroelectric components can be obtained by substituting these functions into the general solutions after determining the constants by the interface compatibility conditions and the mechanical, electric and thermal equilibrium conditions of a cylinder containing the point heat source. Numerical examples are presented at last. All stress components, electric displacement components, and temperature increment are shown graphically by contours.

2. General solutions for the transversely isotropic pyroelectric material

In Cartesian coordinate (x, y, z) with xoy plane parallel with the plane of isotropy, the constitutive relations of transversely isotropic pyroelectric material are

$$
\begin{aligned}
\sigma_x &= c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \Phi}{\partial z} - \lambda_{11}\theta, \\
\sigma_y &= c_{12}\frac{\partial u}{\partial x} + c_{11}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \Phi}{\partial z} - \lambda_{11}\theta, \\
\sigma_z &= c_{13}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + c_{33}\frac{\partial w}{\partial z} + e_{33}\frac{\partial \Phi}{\partial z} - \lambda_{33}\theta, \\
\tau_{yz} &= c_{44}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) + e_{15}\frac{\partial \Phi}{\partial y}, \\
\tau_{zx} &= c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + e_{15}\frac{\partial \Phi}{\partial x}, \\
\tau_{xy} &= c_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),
\end{aligned}
\tag{1a}
$$