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Efficiency of a stochastic restricted two-parameter estimator in linear regression

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ABSTRACT

Sakallıoğlu and Kaçiranlar (2008) proposed an estimator, two-parameter estimator, as an alternative to the ordinary least squares, the ordinary ridge and the Liu estimators in the presence of multicollinearity. In this paper, we introduce a new class estimator by combining the ideas underlying the mixed estimator and the two-parameter estimator when stochastic linear restrictions are assumed to hold. The necessary and sufficient conditions for the superiority of the new estimator over the two-parameter estimator, modified mixed estimator and stochastic restricted two-parameter estimator Yang and Wu (2012) are derived by the matrix mean square error criterion. Furthermore, selections of the biasing parameters are discussed and two numerical examples and a Monte Carlo simulation are given to evaluate the performance of mentioned estimators in the theoretical results.

1. Introduction

Let us consider the linear regression model

$$y = X\beta + e, \quad (1)$$

where $y = (y_1, \ldots, y_n)'$ is a random vector of response variables with mean $E(y) = X\beta$ and covariance matrix $Cov(y) = \sigma^2 I_n$, $X = (x_1, \ldots, x_n)'$ is an $n \times p$ regressor matrix of full column rank with $x_i = (x_{i1}, \ldots, x_{ip})'$ for $i = 1, \ldots, n$, β is a $p \times 1$ vector of unknown regression coefficients, e is an $n \times 1$ vector of disturbances.

The ordinary least (OLS) estimator for regression corresponds to minimizing the sum of squared deviations objective

$$\Psi_1 = (y - X\beta)'(y - X\beta) = y'y - 2\beta'X'y + \beta'X'X\beta, \quad (2)$$

and the objective function (2) minimized by the vector β we have the OLS estimator as

$$\hat{\beta} = (X'X)^{-1}X'y, \quad (3)$$

which plays an important role in regression analysis theory. However, the OLS estimator is unstable and often gives misleading information in the presence of multicollinearity. In order to deal with the multicollinearity, Hoerl and Kennard [1] proposed the ordinary ridge (OR) estimator

$$\hat{\beta}(k) = (X'X + kI)^{-1}X'y, \quad k > 0. \quad (4)$$

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And Liu [2] proposed the Liu estimator which is combined the Stein estimator with the OR estimator

$$\hat{\beta}(d) = (X'X + I)^{-1}\left(X'y + d\hat{\beta}\right), \quad 0 < d < 1 \tag{5}$$

and both of the OR estimator and Liu estimators have become the most common methods to overcome the weakness of OLS estimator. Beside the two estimators, some other biased estimators as ones of remedies were put forward in the literature such as ridge-2 estimator [3], $r-k$ class estimator [4–5], $r-d$ class estimator [6], two-parameter estimator by Sakallıoğlu and Kaçiranlar [7] and alternative two-parameter estimator by Özkale and Kaçiranlar [8].

Another way of solving the muticollinearity problem is to consider parameter estimation with some additional information on the unknown parameters such as the exact or stochastic restrictions [9]. Mostly, the interest has centered around the estimators when exact restrictions assumed to hold (e.g., [10–15]), and there are relatively less results for the estimators with stochastic restrictions. However, as pointed out by Arashi and Tabatabaey [16], exact restrictions are often not suitable in many applied work, involving economic relations, industrial structures, production planning, etc. Meanwhile, stochastic uncertainty occurs in specifying linear programming due to economic and financial studies. In addition, there is also prior information from a previous sample which usually makes some relations through stochastic subspace restrictions. Therefore, we deal with stochastic restrictions in this article.

Let us be given some prior information about β in the form of a set of m independent stochastic linear restrictions as follows:

$$r = R\beta + \varepsilon, \tag{6}$$

where R is a $m \times p$ known matrix with $rank(R) = m$, ε is a $m \times 1$ vector of disturbances with expectation 0 and covariance matrix $\sigma^2 W$, W is assumed to be known and positive definite, the $m \times 1$ vector r can be interpreted as a stochastic known vector. Further it is also assumed that ε is stochastically independent of e.

In the method of mixed estimation as suggested by Theil and Goldberger [17], by unifying the linear model (1) subject to the stochastic linear restrictions (6), we have

$$\tilde{y} = \tilde{X}\beta + \tilde{e}, \tag{7}$$

where

$$\tilde{y} = \begin{pmatrix} y \\ r \end{pmatrix}, \quad \tilde{X} = \begin{pmatrix} X \\ R \end{pmatrix}, \quad \tilde{e} = \begin{pmatrix} e \\ \varepsilon \end{pmatrix}, \quad Cov(\tilde{e}) = \sigma^2\Sigma, \quad \Sigma = \begin{pmatrix} I_n & 0 \\ 0 & W \end{pmatrix}.$$

Since W is positive definite matrix, we know that Σ is also a positive definite matrix. The ordinary mixed estimator of β may be obtained by minimizing

$$\Psi_2 = (\tilde{y} - \tilde{X}\beta)'\Sigma^{-1}(\tilde{y} - \tilde{X}\beta),$$

with respect to β, which is given by

$$\hat{\beta}_{OME} = \left(S + R'W^{-1}R\right)^{-1}\left(X'y + R'W^{-1}r\right) = \hat{\beta} + S^{-1}R'\left(W + R'S^{-1}R\right)^{-1}(r - R\hat{\beta}), \tag{8}$$

where $S = X'X$ and $\hat{\beta}$ is the OLS estimator in (3).

In order to solve the multicollinearity problem, Sakallıoğlu and Kaçiranlar [7] consider the following function

$$\Psi_3 = (y - X\beta)'(y - X\beta) + (\beta - d\hat{\beta}(k))'(\beta - d\hat{\beta}(k)),$$

where d is constant. Then, the two-parameter (TP) estimator [7] is given by

$$\hat{\beta}(k,d) = (S + I)^{-1}\left(X'y + d\hat{\beta}(k)\right). \tag{9}$$

Yang and Wu [18] proposed a stochastic restricted two-parameter estimator, which is obtained by replacing the OLS estimator (3) in Eq. (8) by the two-parameter estimator in Eq. (9),

$$\hat{\beta}_{SR}(k,d) = \hat{\beta}(k,d) + S^{-1}R'\left(W + R'S^{-1}R\right)^{-1}(r - R\hat{\beta}(k,d)). \tag{10}$$

The purpose of this article is to find a better estimator to overcome the multicollinearity in linear regression when stochastic restrictions are assumed to hold. In order to derive the alternative stochastic restricted two-parameter estimator, we propose the modified stochastic restricted two-parameter (MSRTP) estimator by augmenting the equation $d\hat{\beta}(k) = \beta + \varepsilon'$ to Eq. (7) and we can get

$$\Psi_4 = \sigma^2(\tilde{y} - \tilde{X}\beta)'\sigma^{-2}\Sigma^{-1}(\tilde{y} - \tilde{X}\beta) + (\beta - d\hat{\beta}(k))'(\beta - d\hat{\beta}(k)),$$

which combines the TP estimator and ordinary mixed estimator, where d are constant. Differentiation of this function Ψ_4 with respect to β lead to the following normal equations: