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Tension spline method for solution of non-linear Fisher equation

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ABSTRACT

Tension spline method is proposed for the non-linear Fisher equation with initial-boundary values. The three time-level implicit method based on the non-polynomial cubic tension spline is developed for the solution of the non-linear reaction–diffusion equation. The method involves the parameters and its order can be increased from $O(k^2 + k^2h^2 + h^2)$ to $O(k^2 + k^2h^2 + h^4)$ by an appropriate choice of the parameters. The stability of proposed method is analyzed. Finally, numerical results are presented to demonstrate the accuracy and efficiency of this method.

1. Introduction

The problems of the propagation of non-linear waves have fascinated scientists over the years [11,21]. Non-linear waves have been described by modern theories and have been coherent structured in a diverse variety of fields, including plasma, atmosphere and oceans, random media, energy particle physics, combustion, heat and mass transfer, biology, animal dispersal, chemical reactions, non-linear electrical circuits and general relativity and recently it has been acknowledged in the chemical biological and physical communities that the reaction–diffusion equation plays an important role in dissipative dynamical systems. When reaction kinetics and diffusion are coupled, traveling wave of chemical concentration can effect a biochemical change much faster than straight diffusional processes. Usually result of this reaction is reaction–diffusion equation which is one dimensional space:

$$u_t = zu_{xx} + f(u), \quad u(x, t_0) = \phi(x), \quad x \in [L_0, L_1], \tag{1.1}$$

$$u(L_0, t) = P_0(t), \quad u(L_1, t) = P_1(t) \quad t_0 \leqslant t, \tag{1.2}$$

where z is the diffusion coefficient and $f(u)$ represents the kinetics, when

$$f(u) = \alpha u(1 - u^\beta). \tag{1.3}$$

Reaction–diffusion equation is called Fisher equation as a model for the propagation of a mutant gene, with u denoting the density of a advantageous. This equation is encountered in chemical kinetics [13] and population dynamics which includes problems such as non-linear evolution of a population in a nuclear reaction. The exact solution of Eqs. (1.1) and (1.2) for $\beta = 1$ has been obtained by Wang [20] as

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$$u(x,t)=\frac{1}{\left(1+e^{x\sqrt{\frac{z}{6}-\frac{5}{6}zt}}\right)^2}, \tag{1.4}$$

and Eqs. (1.1) and (1.2) place in its simplest form as:

$$u_t = zu_{xx} + \alpha u(1-u), \tag{1.5}$$

where $u(x,t)$ denotes concentration of a fluid or bacteria or a particular biological cell depending upon the nature of the model. General form of Eq. (1.5) is known as Kolmogorov–Petrovskii–Piscounov equation [12] given by

$$u_t = zu_{xx} + f(u),$$

where $f(u)$ is a sufficiently smooth function of u.

Reduces to the well-known Huxley equation when $f(u)$ is a polynomial of u in order three and is given by

$$u_t = zu_{xx} + u^2(1-u). \tag{1.6}$$

This equation has been studied for neural model by Hodgkin and Huxley [8] and Kolmogorov [12]. Some numerical methods such as: quasi-linearization method, Crank–Nicolson formulation, implicit formulation, predictor–corrector explicit method, time-linearization method,..., are used for solving the Fisher equation. These method are compared with each other, by Aggarwal et al. [1]. In [2] the exact solution of generalized Fisher equation has been studied, also in [4] has been tried to find exact and numerical solutions of the generalized Fisher equation. A complete description of lie paint symmetry methods to exploit the invariance properties of the partial differential equations (PDEs) can be found in [9,15,19,22–29]. These methods applied for Fisher equation in [21]. This equation was first studied by Fisher [6,7]. Homotopy perturbation method is used for Fisher's equation and its generalized [14]. Author's studied variational method for Fishers' equation [4] and employed a modified of variational iteration method for generalized Fisher equation [9]. There has been a wide variety of numerical methods, such as finite difference techniques, finite element methods, spectral techniques, adaptive and non-adaptive algorithms, collocation methods,.. which have been developed for its numerical solution [10,11,18,21,30]. Among the most recent numerical techniques for Eqs. (1.1) and (1.2) it is worth mentioning non-standard finite difference methods [11,13], hybrid boundary integral procedure [12]. The nodal integral scheme [4] and piecewise hybrid analytical–numerical algorithms[9]. The paper has been organized as follows: In Section 2, we study formulation and the tension spline method for Eqs. (1.1) and (1.2) and the simplest one, Eq. (1.5). In Section 3, stability and convergence analysis of the Spline difference method ($S.D.M$) are presented. Numerical results are brought in Section 4. Summary of the main conclusion puts at the end of paper. Three time level implicit method by using the non-polynomial cubic tension spline function has been developed for solving Eqs. (1.1) and (1.2). The method involves some parameters and its order can be increased from $o(k^2+k^2h^2+h^2)$ to $o(k^2+k^2h^2+h^4)$ by an appropriate choice of the parameters.

2. Formulation of the Spline difference method:

Considering Eqs. (1.1) and (1.2) in domain $[L_0,L_1]\times[t_0,T]$ which is divided to an $n\times m$ mesh with the step size $h=\frac{L_1-L_0}{n}$ in X direction and $k=\frac{T-t_0}{m}$ in time direction. We define the set of $\psi_h=\{X_L=L_0+lh, l=0(1)n\}$ and $\psi_k=\{t_j=t_0+jk,\ j=0(1)m\}$, in which n and m are integers. A function as $s(x)\in C^2[L_0,L_1]$ interpolates $u(x)$ at the knots $x_l,\ l=0,1,\ldots,n$ which depends on a parameter $\omega>0$. When $\omega\to 0$, $s(x)$ reduces to a cubic spline (non-polynomial spline). By defining $s(x)$ in the interval $[x_l,x_{l+1}],\ l=0,\ldots,n-1$,

$$S(x)=a_l+b_l(x-x_0)+c_l(e^{\omega(x-x_l)}-e^{-\omega(x-x_l)})+d_l(e^{\omega(x-x_l)}+e^{-\omega(x-x_l)}),\quad l=0,\ldots,n, \tag{2.7}$$

where a_l, b_l, c_l and d_l are un-known coefficients and ω is the free parameter. If we set:

$$S(x)=u_l,\quad S(x_{l+1})=u_{l+1},\quad S''(x_l)=M_l,\quad S''(x_{l+1})=M_{l+1}, \tag{2.8}$$

we can develop explicit expressions for the four coefficients in (2.7) as follow:

$$a_l=u_l-\frac{M_l}{\omega^2},\quad b_l=\frac{M_l-M_{l+1}+\omega^2(u_{l+1}-u_l)}{\omega\theta},\quad c_l=\frac{2M_{l+1}-(e^\theta+e^{-\theta})M_l}{2\omega^2(e^\theta-e^{-\theta})},\quad d_l=\frac{M_l}{2\omega^2},$$

where $\theta=\omega h$. Now we use the continuity of its first derivative at mesh point (x_l,u_l), so the following equation is obtained:

$$u_{l+1}-2u_l+u_{l-1}=h^2[\alpha M_{l+1}+2\beta M_l+\alpha M_{l-1}], \tag{2.9}$$

where

$$\alpha=\frac{1}{\theta^2}\left(1-\frac{2\theta}{(e^\theta-e^{-\theta})}\right),\quad \beta=\frac{1}{\theta^2}\left(\frac{\theta(e^\theta+e^{-\theta})}{(e^\theta-e^{-\theta})}-1\right).$$

Non-polynomial $S(x)$ in (2.7) reduces into ordinary cubic spline relation in [16] when $\omega\to 0$ then $(\alpha,\beta)\to(\frac{1}{6},\frac{1}{3})$ and the relation (2.9) reduces to the following relation:

$$u_{l+1}-2u_l+u_{l-1}=\frac{h^2}{6}[M_{l+1}+4M_l+M_{l-1}]. \tag{2.10}$$