Applied Mathematics and Computation 247 (2014) 152–161

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Controllability results for a class of fractional semilinear integro-differential inclusions via resolvent operators

V. Vijayakumar [a,*], C. Ravichandran [b], R. Murugesu [c], J.J. Trujillo [d]

[a] Department of Mathematics, Info Institute of Engineering, Kovilpalayam, Coimbatore - 641 107, Tamil Nadu, India
[b] Department of Mathematics, KPR Institute of Engineering and Technology, Arasur, Coimbatore - 641 407, Tamil Nadu, India
[c] Department of Mathematics, SRMV College of Arts and Science, Coimbatore - 641 020, Tamil Nadu, India
[d] Universidad de La Laguna, Departamento de Análisis Matemático, 38271 La Laguna, Tenerife, Spain

ARTICLE INFO

Keywords:
Controllability
Fractional integro-differential inclusions
Multivalued map
Resolvent operators
Bohnenblust–Karlin's fixed point theorem

ABSTRACT

In this paper, we consider a class of fractional integro-differential inclusions in Banach spaces. This paper deals with the controllability for fractional integro-differential control systems. First, we establishes a set of sufficient conditions for the controllability of fractional semilinear integro-differential inclusions in Banach spaces via resolvent operators. We use Bohnenblust–Karlin's fixed point theorem to prove our main results. Further, we extend the result to study the controllability concept with nonlocal conditions. An example is also given to illustrate our main results.

1. Introduction

During last decades, fractional differential equations have been used as effective mathematical models of the dynamics of many different processes through anomalous media in applied areas, for instance, in wave propagation, electromagnetism, heat transfer, biology, signal processing, robotics, genetic algorithms, telecommunications, control theory and so on. Moreover, many real complex physical systems have been represented more accurately through fractional derivative formulation. In the recent years, there has been a significant development in ordinary and partial differential equations involving fractional differential operators, see, for example, the monographs by Miller and Ross [34], Kilbas et al. [24], Lakshmikantham et al. [27], Uchaikin [41], the papers [1,2,37,30] and the references therein. On the other hand, the fractional differential inclusions arise in the mathematical modeling of certain problems in economics, optimal controls, etc., so the problem of existence of solutions of differential inclusions and fractional differential inclusions has been studied by several authors for different kind of problems (see, for example, [2,7,8,13,14,39,43,45,48,50] and references therein).

Controllability is one of the primary concept in mathematical control theory. The concept of controllability plays a major role in both finite and infinite dimensional spaces, that is, systems represented by ordinary differential equations and partial differential equations respectively. So it is natural to extend this concept to dynamical systems represented by fractional differential equations. Many partial differential, fractional differential equations and integro-differential equations can be expressed as fractional differential and integro-differential equations in some Banach spaces [4–7,13,14,16,17,29,31–33,36,38,25,26,39,40,42–46,48,49].

Recently, Agarwal et al. [3] proved the existence and qualitative properties of an α resolvent operator for an abstract fractional integro-differential problem of the form

* Corresponding author.
E-mail addresses: vijaysarovel@gmail.com (V. Vijayakumar), ravibirthday@gmail.com (C. Ravichandran), arjhunmurugesh@gmail.com (R. Murugesu), jtrujill@ullmat.es (J.J. Trujillo).

http://dx.doi.org/10.1016/j.amc.2014.08.080
0096-3003/

$$\begin{cases} D_t^\alpha x(t) = Ax(t) + \int_0^t G(t-s)x(s)ds + f(t), & t \in (0,b] \\ x(0) = x_0, \quad x'(0) = 0, \end{cases}$$

where $\alpha \in (1,2)$. Since the appearance of such manuscript, several papers have addressed the issue of existence results and controllability results for fractional differential equations via α resolvent operators [15,19–21,43,44], although many problem remain open on this interesting topic.

Motivated by the above works, this paper establishes a set of sufficient conditions for the controllability of fractional semilinear integro-differential inclusions in Banach spaces of the form

$$\begin{cases} D_t^\alpha x(t) \in Ax(t) + \int_0^t G(t-s)x(s)ds + Bu(t) + F(t,x(t)), & t \in I = [0,b] \\ x(0) = x_0, \quad x'(0) = 0, \end{cases} \tag{1}$$

where $\alpha \in (1,2)$; A, $(G(t))_{t\geqslant 0}$ are closed linear operators defined on a common domain which is dense in a Banach space X, ${}^cD^\alpha_{0+t}h(t) = D_t^\alpha h(t)$ represent the well known fractional Caputo derivative of order $n-1 < \alpha < n$ of a suitable function h is defined by

$$D_t^\alpha h(t) = \int_0^t g_{n-\alpha}(t-s)\frac{d^n}{ds^n}h(s)ds,$$

where n is the smallest integer greater than or equal to α and $g_\beta(t) := \frac{t^{\beta-1}}{\Gamma(\beta)}$, $t > 0$, $\beta > 0$. The control function $u(\cdot) \in L^2(I,U)$, a Banach space of admissible control functions. Further, B is a bounded linear operator from U to X, and $F : I \times X \to 2^X \setminus \{\emptyset\}$ is a nonempty, bounded, closed and convex multivalued map.

To the best of our knowledge, the study of the controllability of fractional semilinear integro-differential inclusions in Banach spaces of the form (1), is an untreated topic in the literature, this will be the main motivation of our paper. This paper is organized as follows. In Section 3, we establish a set of sufficient conditions for the controllability of fractional semilinear integro-differential inclusions in Banach spaces. In Section 4, we establish a set of sufficient conditions for the controllability of fractional semilinear integro-differential inclusions with nonlocal conditions. An example is presented in Section 5 to illustrate the theory of the obtained results.

2. Preliminaries

In what follows, we recall some definitions, notations and results that we need in the sequel. Throughout this paper, $(X, \|\cdot\|)$ is a Banach space and A, $G(t)$, for $t \geqslant 0$, are closed linear operators defined on a common domain $\mathcal{D} = D(A)$ which is dense in X. The notation $[D(A)]$ represents the domain of A endowed with the graph norm. Let $(Z, \|\cdot\|_Z)$ and $(W, \|\cdot\|_W)$ be Banach spaces. In this paper, the notation $\mathcal{L}(Z,W)$ stands for the Banach space of bounded linear operators from Z into W endowed with the uniform operator topology and we abbreviate this notation to $\mathcal{L}(Z)$ when $Z = W$. Furthermore, for appropriate functions $K : [0,\infty) \to Z$ the notation $\hat{K}$ denotes the Laplace transform of K. The notation, $B_r[x,Z]$ stands for the closed ball with center at x and radius $r > 0$ in Z. On the other hand, for a bounded function $x : [0,a] \to Z$ and $b \in [0,a]$, the notation $\|x\|_{Z,b}$ is defined by

$$\|x\|_{Z,b} = \sup\{\|x(s)\|_Z : s \in [0,b]\}$$

and we simplify this notation to $\|x\|_b$ when no confusion about the space Z arises.

To obtain our results, we assume that the abstract fractional integro-differential problem

$$D_t^\alpha x(t) = Ax(t) + \int_0^t G(t-s)x(s)ds, \tag{2}$$

$$x(0) = z \in X, \quad x'(0) = 0, \tag{3}$$

has an associated α-resolvent operator of bounded linear operators $(\mathcal{R}_\alpha(t))_{t\geqslant 0}$ on X.

Definition 2.1. A one-parameter family of bounded linear operators $(\mathcal{R}_\alpha(t))_{t\geqslant 0}$ on X is called an α-resolvent operator of (2)–(3) if the following conditions are verified.

(a) The function $\mathcal{R}_\alpha(\cdot) : [0,\infty) \to \mathcal{L}(X)$ is strongly continuous and $\mathcal{R}_\alpha(0)x = x$ for all $x \in X$ and $\alpha \in (1,2)$.
(b) For $x \in D(A)$, $\mathcal{R}_\alpha(\cdot)x \in C([0,\infty),[D(A)]) \bigcap C^1([0,\infty),X)$, and

$$D_t^\alpha \mathcal{R}_\alpha(t)x = A\mathcal{R}_\alpha(t)x + \int_0^t G(t-s)\mathcal{R}_\alpha(s)xds, \tag{4}$$

$$D_t^\alpha \mathcal{R}_\alpha(t)x = \mathcal{R}_\alpha(t)Ax + \int_0^t \mathcal{R}_\alpha(t-s)G(s)xds, \tag{5}$$

for every $t \geqslant 0$.