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Impulsive neutral stochastic functional integro-differential equations with infinite delay driven by fBm ☆

Yong Ren [a,*], Xing Cheng [a], R. Sakthivel [b]

[a] Department of Mathematics, Anhui Normal University, Wuhu 241000, China
[b] Department of Mathematics, Sungkyunkwan University, Suwon 440-746, South Korea

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ABSTRACT

In this paper, we study a class of impulsive neutral stochastic functional integro-differential equations with infinite delay driven by a standard cylindrical Wiener process and an independent cylindrical fractional Brownian motion (fBm) with Hurst parameter $H \in (1/2, 1)$ in the Hilbert space. We prove the existence and uniqueness of the mild solution for this kind of equations with the coefficients satisfying some non-Lipschitz conditions, which include the classical Lipschitz conditions as special case. An example is provided to illustrate the theory. Some well-known results are generalized and extended.

1. Introduction

In this paper, we are concerned with the existence and uniqueness of the mild solution for the following impulsive neutral stochastic functional integro-differential equations with infinite delay driven by a standard Wiener process and an independent fractional Brownian motion

$$\begin{cases} \mathrm{d}[x(t)+g(t,x_t)] = A[x(t)+g(t,x_t)]\mathrm{d}t + \left[\int_0^t B(t-s)[x(s)+g(s,x_t)]\mathrm{d}s + f(t,x_t)\right]\mathrm{d}t + h(t,x_t)\mathrm{d}w(t) + \sigma(t)\mathrm{d}B_Q^H(t), \quad t \in J := [0,T],\ t \neq t_k, \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = I_k(x(t_k)), \quad k = 1,2,\dots,m, \\ x(t) = \varphi(t) \in D^B_{\mathcal{F}_0}((-\infty,0],X), \quad t \in J_0 := (-\infty,0], \end{cases} \tag{1.1}$$

where $x(\cdot)$ takes value in a real separable Hilbert space X with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$; A is the infinitesimal generator of a strongly continuous semigroup $(S(t))_{t\geqslant 0}$ on X with domain $D(A)$; $B(t)$ is a closed linear operator on X with domain $D(B) \supset D(A)$ which is independent of t; B^H is a fractional Brownian motion with Hurst parameter $H \in (1/2,1)$ and $\{w(t) : t \in J\}$ is a standard Wiener process on a real and separable Hilbert space Y; $T \geqslant 0$ is a fixed real number. In the sequel, let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space and for $t \geqslant 0$, $\mathcal{F}_t$ denote the σ-field generated by $\{B_Q^H(s), w(s), s \in [0,t]\}$ and the $\mathbb{P}$-null sets. Further, we assume that w and B_Q^H are independent. Let $L(Y,X)$ be the space of all bounded, continuous and linear operators from Y into X. Assume that $g, f : [0,+\infty) \times \widehat{D} \to X$, $h : [0,+\infty) \times \widehat{D} \to L_2^0(Y,X)$, $\sigma : [0,+\infty) \to L_Q^0(Y,X)$ are appropriate functions. Here, $\widehat{D} = D((-\infty,0],X)$ denotes the family of all right piecewise continuous functions with left-hand limit φ from $(-\infty,0]$ to X. For equations with infinite delay, the segment $x_t : (-\infty,0] \to X$ is defined by $x_t(\theta) = x(t+\theta)$ for $t \geqslant 0$ belongs to the phase space $\widehat{D}$. The space L_2^0 and L_Q^0 will be defined in the next section. Here, $I_k \in C(X,X)$ $(k = 1,2,\dots,m)$ are bounded

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* Corresponding author.
E-mail addresses: renyong@126.com, brightry@hotmail.com (Y. Ren).

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functions and the fixed times t_k satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_m < T, x(t_k^+)$ and $x(t_k^-)$ denote the right and left limits of $x(t)$ at time t_k. And $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ represents the jimp in the state x at time t_k, where I_k determines the size of the jump. The initial data $\varphi = \{\varphi(t) : -\infty < t \leqslant 0\}$ is an $\mathcal{F}_0$-measurable, $\widehat{D}$-valued stochastic process independent of the wiener process w and the fBm B_Q^H with finite second moment.

In the past decades, the theory of impulsive integro-differential equations has became an active area of investigation due to their applications in the fields such as mechanics, electrical engineering, medicine biology, ecology and so on. One can see [7,17] and the references therein. Several authors have established the existence results of mild solutions for these equations (see [2,3,15,18] and the references therein).

For the potential applications in telecommunications networks, finance markets, biology and other fields ([8,10,13]), stochastic differential equations driven by fractional Brownian motion (fBm) have attracted researchers' great interest. Especially, Duncan et al. [9] proved the existence and uniqueness of a mild solution for a class of stochastic differential equations in a Hilbert space with a standard, cylindrical fBm with the Hurst parameter in the interval (1/2, 1). Moreover, Maslowski and Nualart [14] established the existence and uniqueness of a mild solution for nonlinear stochastic evolution equations in a Hilbert space driven by a cylindrical fBm under some regularity and boundedness conditions on the coefficients. very recently, Caraballo et al. [6] investigated the existence and uniqueness of mild solutions to stochastic delay evolution equations driven by a fBm with Hurst parameter $H \in (1/2, 1)$. An existence and uniqueness result of mild solutions for a class of neutral stochastic differential equation with finite delay, driven by a fBm in a Hilbert space has been recently established in Boufoussi and Hajji [5].

To the best of our knowledge, there is no work on the impulsive neutral stochastic functional integro-differential equations driven by a standard cylindrical Wiener process and an independent cylindrical fBm with Hurst parameter $H \in (1/2, 1)$ in the Hilbert space. Motivated by the previously mentioned paper, in this work, we aim to study this interesting problem. We prove the existence and uniqueness of the mild solution for this kind of equations with the coefficients satisfying some non-Lipschitz conditions, which include the classical Lipschitz condition as special case. An example is provided to illustrate the theory. We would like to mention we have extended and generalized the results appeared in Anguraj and Vinodkumar [1], Boufoussi and Hajji [5], Caraballo et al. [6], Ren and Xia [19].

The paper is organized as follows. In Section 2, we introduce some preliminaries. Section 3 proves the existence and uniqueness of a mild solution for the system (1.1). An example is provided in the last section to illustrate the theory.

2. Preliminaries

In this section, we provide some preliminaries needed to establish our main results. For details of this section, we refer the reader to [5,6,11] and the references therein. Throughout this paper $(X, \|\cdot\|, \langle\cdot,\cdot\rangle)$ and $(Y, \|\cdot\|_Y, \langle\cdot,\cdot\rangle_Y)$ are two real separable Hilbert spaces. The notation $L^2(\Omega, X)$ stands for the space of all X-valued random variables x such that $E\|x\|^2 = \int_\Omega \|x\|^2 \, d\mathbb{P} < \infty$. For $x \in L^2(\Omega, X)$, let $\|x\|_2 = \left(\int_\Omega \|x\|^2 \, d\mathbb{P}\right)^{\frac{1}{2}}$. It is easy to check that $L^2(\Omega, X)$ is a Hilbert space equipped with the norm $\|\cdot\|_2$. Let $L(Y, X)$ denotes the space of all bounded linear operators form Y to X, we abbreviate this notation to $L(Y)$ whenever $X = Y$ and $Q \in L(Y)$ represents a non-negative self-adjoint operator.

Let Y_0 be an arbitrary separable Hilbert space and $L_2^0 = L^2(Y_0, X)$ be a separable Hilbert space with respect to the Hilbert–Schmidt norm $\|\cdot\|_{L_2^0}$. Let $L_Q^0(Y, X)$ be the space of all $\psi \in L(Y, X)$ such that $\psi Q^{\frac{1}{2}}$ is a Hilbert–Schmidt operator. The norm is given by $\|\psi\|^2_{\mathcal{L}_Q^0} = \left\|\psi Q^{\frac{1}{2}}\right\|^2 = tr(\psi Q \psi^*)$. Then ψ is called a Q-Hilbert–Schmidt operator from Y to X. In the sequel, $L_2^0(\Omega, X)$ denotes the space of $\mathcal{F}_0$-measurable, X-valued and square integrable stochastic processes.

2.1. fractional Brownian motion and the phase space

Now, we recall some basic knowledge on the fBm as well as the Wiener integral with respect to it. For more details, one can see Caraballo et al. [6] and Nualart [16]. Consider a time interval $[0, T]$ with arbitrary fixed horizon T and let $\{\beta^H(t), t \in [0, T]\}$ be a one-dimensional fBm with Hurst parameter $H \in (1/2, 1)$. By definition, it means that β^H is a continuous centered Gaussian process with the covariance function:

$$R_H(r, s) = \frac{1}{2}\left(s^{2H} + r^{2H} - |s - r|^{2H}\right).$$

Further, β^H has the following Wiener integral representation;

$$\beta^H(t) = \int_0^t K_H(t, s) \, d\beta(s),$$

where $\beta = \{\beta(t) : t \in [0, T]\}$ is a Wiener process and $K_H(t, s)$ is the kernel given by

$$K_H(t, s) = c_H s^{\frac{1}{2}-H} \int_s^t (y - s)^{H-3/2} y^{H-1/2} \, du \quad \text{for } t > s,$$

here $c_H = \sqrt{\frac{H(2H-1)}{B(2-2H, H-\frac{1}{2})}}$ with $B(\cdot)$ represents the Beta function. We take $K_H(t, s) = 0$ if $t \leqslant s$.