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# New alternatives to ratio estimators of population variance in sample surveys

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ABSTRACT

In this paper we propose alternative estimators to ratio estimators given by Kadilar and Cingi (2006) and a class of estimators for population variance using the same approach adopted by Srivenkataramana (1980). Conditions under which the proposed estimators are more efficient than usual unbiased estimator, Isaki (1983) estimator, Biradar and Singh (1994) estimator and the estimators given by Kadilar and Cingi (2006) are obtained. The exact expressions for biases and mean squared errors of the proposed estimators are obtained. An empirical study has been carried out to demonstrate the performance of the suggested estimators. Also following Searls (1964), generalized version of the proposed class of estimators is also studied with its properties.



## 1. Introduction

Auxiliary information is often used in sample surveys to increase the precision of the estimators. For estimating the population parameters such as population mean, population variance etc. several authors have often used information on different parameters such as population mean, coefficient of variation, coefficient of kurtosis and coefficient of skewness of the auxiliary variate. Many researchers including Searl [9], Sisodia and Dwivedi [14], Pandey and Dubey [8], Singh and Tailor [10], Singh et al. [13], Tailor and Tailor [21], Tailor and Sharma [20], Tailor et al. [22], Tailor et al. [23], Solanki et al. [15] and Tailor and Lone [18] have paid their attention towards the improved estimation of population mean of the variable under study. In spite the estimation of population mean of the study variate, estimation of the population variance using supplementary information on the auxiliary variate has also attracted the attention of the survey statisticians. The problem of estimating the population variance using auxiliary information has been discussed by various authors including [5,24–26,1–3,17,6,12,19] etc.

Let $U = (U_1, U_2, \ldots, U_N)$ be a finite population of $N$ units. Let $(y, x)$ be the variate observed on $U_i (i = 1, 2, 3, \ldots, N)$. The values of auxiliary variate $x$ are known for all units of the population. A sample of size $n$ is drawn from population $U$ using simple random sampling without replacement. Let us define

$$S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2 = \frac{N}{(N-1)}\sigma_y^2 = \frac{N}{(N-1)}\mu_{20},$$

$$S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2 = \frac{N}{(N-1)}\sigma_x^2 = \frac{N}{(N-1)}\mu_{02},$$

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$$\overline{Y} = N^{-1}\sum_{i=1}^{N} y_i, \quad \overline{X} = N^{-1}\sum_{i=1}^{N} x_i, \quad \beta_2(x) = \frac{\mu_{04}}{\mu_{02}^2}, \quad \beta_2(y) = \frac{\mu_{40}}{\mu_{20}^2},$$

$$\beta_2^*(y) = [K\beta_2(y) - M], \quad \beta_2^*(x) = [K\beta_2(x) - M].$$

$$\mu_{ab} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \overline{Y})^a (x_i - \overline{X})^b; \quad (a, b) \text{ being non negative integers;}$$

$$K = \frac{(N-1)(Nn - N - n - 1)}{(n-1)N(N-3)}, \quad M = \frac{(N^2 n - 3N^2 + 6N - 3n - 3)}{(n-1)N(N-3)},$$

$$E(s_y^2 s_x^2) = S_y^2 S_x^2\left[1 + \frac{(N-n)}{n(N-2)}(Kl + 2B\rho^2 - D)\right], \tag{1.1}$$

$$E\left[s_y^2(\bar{x} - \overline{X})^2\right] = \frac{(N-n)}{n^2(N-1)}\left[\frac{(N^2 - 2Nn + N)}{(N-2)(N-3)}\mu_{22} + \frac{N(Nn - 2n - N + 1)}{(N-2)(N-3)}\mu_{20}\mu_{02} - 2\frac{N(N-n-1)}{(N-2)(N-3)}\mu_{11}^2\right]. \tag{1.2}$$

where $l = \frac{\mu_{22}}{\mu_{20}\mu_{02}}$ and $\rho = \frac{\mu_{11}}{(\mu_{20}\mu_{02})^{1/2}}$.

For the sample, we define $s_y^2$ and $s_x^2$, the variances of $y$ and $x$ respectively as

$$s_y^2 = (n-1)^{-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 \quad \text{and} \quad s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

Further we define

$$s_x^{*2} = \left[\frac{(N-1)S_x^2 - (n-1)s_x^2}{(N-n-1)} - \frac{nN(\bar{x} - \overline{X})^2}{(N-n-1)(N-n)}\right] \tag{1.3}$$

which is also an unbiased estimator of $S_x^2$ based on unobserved units. when the population variance $S_x^2$ of the auxiliary variates $x$ is known, Isaki [5] has proposed the estimator for $S_y^2$ as

$$t_1 = s_y^2\left(\frac{S_x^2}{s_x^2}\right), \tag{1.4}$$

The mean squared error of $t_1$ up to the first degree of approximation is given by

$$MSE(t_1) = \frac{(N-n)S_y^4}{(N-2)n}\left[\beta_2^*(y) + \beta_2^*(x) - 2(Kl + 2B\rho^2 - D)\right]. \tag{1.5}$$

Using Srivenkataramana [16] transformation, Biradar and Singh [2] proposed an alternative to Isaki [5] ratio estimator as

$$t_2 = s_y^2\left(\frac{s_x^{*2}}{S_x^2}\right). \tag{1.6}$$

The mean squared error of $t_2$ up to second order moments is given as

$$MSE(t_2) = \frac{(N-n)}{(N-2)n}S_y^4\left[\left\{\frac{N-n}{N-n-1}\right\}^2\beta_2^*(y) + \left(\frac{n-1}{N-n-1}\right)^2\beta_2^*(x) - 2\frac{(n-1)}{(N-n-1)^2}(N-n+1)(Kl + 2B\rho^2 - D) - \frac{n(N-2)}{(N-n-1)^2(N-n)}\right]. \tag{1.7}$$

when the population coefficient of Kurtosis $\beta_2(x)$, population coefficient of variation $C_x$ along with population variance $S_x^2$ is known, Kadilar and Cingi [6] have proposed the ratio type estimators for $S_y^2$ as

$$t_3 = s_y^2\left(\frac{S_x^2 - C_x}{s_x^2 - C_x}\right), \tag{1.8}$$

$$t_4 = s_y^2\left(\frac{S_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)}\right), \tag{1.9}$$