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Shape-preserving surfaces with constraints on tension parameters

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ABSTRACT

The C^1 rational bi-cubic local interpolation schemes are presented for the shape preservation of convex, monotone and positive surface data. The shape of the surface is controlled locally with the help of eight tension parameters over each rectangular patch. Data dependent constraints are developed on half of the tension parameters to preserve the intrinsic shapes of the surface data. The remaining ones are unconstrained, thus free to be used to obtain the smoothest surface.

1. Introduction

The interpolation techniques which retain the intrinsic properties (positivity, monotonicity, convexity) of data are of great interest in various applications e.g. visualization of probabilistic data, modeling of two-dimensional semi-Lagrangian advection in plane and spherical geometry, designing of very large scale circuit(VLSC) in engineering etc. The ordinary data interpolation techniques join the data smoothly but do not inherit its shapes. The available shape-preserving techniques restrain the derivatives at the knots to interpolate positive, monotone and convex data as positive, monotone and convex curves and surfaces. These derivatives based shape-preserving interpolation techniques work well with certain limitations e.g. the solution of ordinary and partial differential equations cannot be interpolated by these shape-preserving methods. Another approach is to modify the data by introducing new knots between the given data points. Though these schemes preserve the shapes of data, but the computational labour has increased. In this research paper, the rational interpolating function with tension parameters is introduced to preserve the intrinsic shapes of surface data. The tension parameters are converted into shape parameters to obtain the shape-preserving C^1 rational bi-cubic interpolating function for given data with derivatives.

A prevue of the existing shape-preserving interpolation schemes is provided next. Brodlie, Mashwama and Butt [1] developed a positivity preserving bi-cubic Hermite interpolation scheme. The constraints were derived on the nodal first order partial derivatives and mixed partial derivatives to preserve the positivity of regular surface data. Brodlie, Asim and Unsworth [2] used Modified Quadratic Shepard interpolant to preserve the shape of positive scattered data. The quadratic basis functions were positively constrained to interpolate positive data as positive surface. The developed results were further applied to the problem of range restricted interpolation. Luo and Peng [12] developed a C^1 rational spline interpolation scheme for the range restricted interpolation of scattered data. The C^1 rational spline was the convex combination of three cubic Bézier triangular patches. The range restricting constraints were the system of inequalities with Bézier ordinates as

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parameters. Mulansky and Schmidt [14] constructed a C^1 bivariate quadratic spline interpolation scheme for the range restricted interpolation of scattered data. The derived sufficient conditions for the fulfillment of range restrictions resulted in a solvable system of inequalities with gradients as parameters. Since a system of inequalities has infinite many solutions, therefore, the optimal solution was obtained by local and global quadratic programming problems subject to these constraints. In [10], surface data visualization schemes were proposed using rational bi-cubic functions.

Delgado and Peña [5] exemplified that the rational Bézier surfaces on rectangular grid, tensor-product surfaces with rational basis and rational Bézier surfaces on triangular grids are not monotonicity preserving. Floater and Peña [6] studied the different system of basis functions: the Bernstein polynomials, B-splines and the totally positive system (trigonometric and rational basis). It was observed that the tensor-product surfaces based on Bernstein polynomials and B-spline basis preserved the monotonicity of the control net, whereas, the totally positive rational basis function failed to preserve the monotonicity of data. Mainar and Peña [13] identified that the tensor product surfaces with basis from algebraic, trigonometric and hyperbolic polynomials spaces are the only monotonicity preserving surfaces besides tensor product Bézier and tensor product B-spline surfaces. Carlson and Fritsch [3] extended their result of monotone univariate interpolation to monotone bivariate interpolation for data arranged on a rectangular grid. The interpolating function defined in terms of the first order partial derivatives and twists at all grid points. Necessary and sufficient conditions on these derivatives were derived such that the resulting bi-cubic polynomial was monotone on a single rectangular element. Costantini and Fontanella [4] developed a semi-global scheme to preserve the shape of convex data. The drawback of the scheme was that in some rectangular patches, the degree of interpolant becomes too large and the polynomial patches tend to be linear in x and/or y. The resulting surfaces were not always visually pleasing. Fujioka and Kano [7] proposed an optimal spline approximation scheme using normalized B-spline basis. The monotonicity and convexity constraints were applied on the partial derivatives at the knots. Thus the problem was reduced to convex quadratic programming problem subject to monotonicity and convexity constraints. Renka [15] described a Fortran 77 software package for constructing a C^1 convex surface that interpolates arbitrarily distributed convex data sets. The method consists of constructing a data dependent triangulation, computing a set of nodal gradients for which there exists a convex piecewise linear Hermite interpolant of the nodal values and gradient.

In this research paper, firstly we have constructed a C^1 rational bi-cubic interpolating function. It has eight tension parameters in each rectangular patch. Three different set of constraints are developed on half of the tension parameters to preserve positive, monotone and convex shapes of surface data, thus these tension parameters works as shape-preserving parameters. The rest of the tension parameters are unconstrained, hence available as free parameters for shape refinement. The proposed shape-preserving C^1 rational bi-cubic interpolating schemes have the following benefits:

- Brodlie and Butt [1], Luo and Peng [12], Mulansky and Schmidt [14] constrained the nodal derivatives to preserve that positive shape of surface data. The proposed schemes of [1], [12] and [14] are not applicable to data with derivatives. The positivity-preserving scheme of this paper develops restrains on tension-parameters thus can be applied to data with derivatives without any hesitation.
- In [2], the basis functions were constrained to preserve the positive shape of data, whereas, the positivity preserving scheme of this paper has same basis functions for all data type.
- In [5], it was established that surfaces generated with rational basis functions do not preserve the monotone shape of surface data. In this paper, a monotonicity preserving scheme based on rational basis is developed.
- The monotonicity-preserving schemes, Carlson and Fritsch [3] and Fujioka and Kano [7], preserved the monotone shape of data by the modification of partial derivatives computed from data. In Renka [15], the convexity was preserved by the suitable choice of nodal gradients. Unlike [3], [7] and [15], the monotonicity and convexity preserving schemes presented in this paper develops constrains on tension parameters, hence the derivatives values remains unchanged. It makes these schemes suitable for the visualization of data with derivatives.
- Costantini and Fontanella [4] proposed a semi-global shape-preserving scheme, whereas, the shape-preserving schemes developed here are local.
- The rational shape-preserving schemes [9] also developed constraints on parameters to preserve the shapes of data. However, these schemes were not C^1. The shape-preserving schemes which we introduced in this paper are C^1.
- Although the positivity-preserving scheme [11] was C^1 but not local. All the shape-preserving schemes of this paper are local.
- The monotonicity-preserving algorithm [8] was a C^1 rational scheme, where constraints are developed on all the parameters to preserve the monotone shape of data. Hence no parameter is free for shape modification. We have proposed the C^1 local shape-preserving schemes with four free parameters in each rectangular patch.

The study is organized into several sections. The Section 2 reviews the rational cubic function [16]. In Section 3, a C^1 rational bi-cubic interpolating function is constructed. In Section 4, Section 5, Section 6 and Section 7 shape-preserving interpolation schemes are developed. Section 8 concludes the paper.

2. Rational cubic function

In this section, the rational cubic function developed by Sarfraz et al. [16] is outlined.