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Improving the applicability of the secant method to solve nonlinear systems of equations

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ABSTRACT

A modification of the secant method for the approximation of nonlinear system of equations is considered to improve the applicability of the secant method. This modification changes the resolution of a linear system in each step, necessary to apply the secant method, for several matrix multiplications. In this way, the numerical stability of the secant method can be improved. In addition, in this paper, we prove that the modification considered keeps two important properties of the secant method, such as: does not use derivatives in its algorithm and has R-order of convergence at least $\left(1+\sqrt{5}\right)/2$.

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1. Introduction

Many scientific and engineering problems can be written in the form of a nonlinear systems of equations $F(x) = 0$, where F is a nonlinear operator defined on a non-empty open convex subset Ω of $\mathbb{R}^m$ with values in $\mathbb{R}^m$. In general, if the operator F is nonlinear, iterative methods are used to solve $F(x) = 0$.

In this paper, we consider the secant method [1,6,7,12] to approximate a solution of the nonlinear system of equations $F(x) = 0$. It is a one-point iterative process with memory given by the following algorithm:

$$\begin{cases} x_{-1}, x_0 \text{ given}, \\ x_{n+1} = x_n - [x_{n-1}, x_n; F]^{-1} F(x_n), \quad n \geqslant 0, \end{cases} \tag{1}$$

where $[u, v; F], u, v \in \Omega$, is a first order divided difference [11,13], which is a bounded linear operator such that

$$[u, v; F] : \Omega \subset \mathbb{R}^m \longrightarrow \mathbb{R}^m \quad \text{and} \quad [u, v; F](u - v) = F(u) - F(v).$$

Along the paper, we consider the following first order divided difference in $\mathbb{R}^m$: $[u, v; F] = ([u, v; F]_{ij})_{i,j=1}^m$, where

$$[u, v; F]_{ij} = \frac{1}{u_j - v_j}\left(F_i(u_1, \ldots, u_j, v_{j+1}, \ldots, v_m) - F_i(u_1, \ldots, u_{j-1}, v_j, \ldots, v_m)\right),$$

$u = (u_1, u_2, \ldots, u_m)^T$ and $v = (v_1, v_2, \ldots, v_m)^T$.

The interest of using the secant method (1) for solving the nonlinear system of equations $F(x) = 0$ lies in that derivatives of the operator F do not appear in the algorithm, contrary to what occurs in Newton's method [15]. Moreover, this iterative

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process has superlinear convergence [11,13]. However, in the secant method remains an interesting problem to solve. As is the case for most of the iterative processes, its algorithm needs to solve a linear system of equations at each step:

$$\begin{cases} x_{-1}, x_0 \text{ given}, \\ [x_{n-1}, x_n; F](x_{n+1} - x_n) = F(x_n), \quad n \geqslant 0. \end{cases} \tag{2}$$

In order to improve this fact, following the ideas that Moser applied to solve this problem in the case of Newton's method [4], we consider in [5] the following iterative process

$$\begin{cases} x_0, B_0 \text{ given}, \\ x_{n+1} = x_n - B_n F(x_n), \quad n \geqslant 0 \\ B_{n+1} = 2B_n - B_n[x_n, x_{n+1}; F]B_n, \end{cases} \tag{3}$$

where we have changed the resolution of the linear system in every step by making several matrix multiplications. Whereas the operational cost of both processes is equivalent, however the possibility of *ill-conditioned* linear systems appearing will be avoided simply taking appropriate initial matrix B_0 , and therefore this previous algorithm will be able to be stable more easily. The matrix given by the divided difference $[x_{n-1}, x_n; F]$, can be *ill-conditioned* and therefore the linear system of equations previously indicated in the classical secant method should cause instabilities.

In this paper, we study stability, the semilocal convergence and the R-order of convergence of the proposed algorithm (3) and we perform a comparison with the secant method. From this study, we can conclude that the considered method (3) improves the applicability of the secant method.

To finish the paper, we consider more numerical experiments including the approximation of a boundary problem associated to the Lorenz system.

2. On the stability of the secant method

We consider the academic system of equations $F(x, y) = (0, 0)$ given by:

$$\begin{aligned} &\left(2x - \frac{x^2}{\epsilon}\right) + \left(y - \frac{y^2}{2\epsilon}\right) = 0, \\ &x + y = 0. \end{aligned} \tag{4}$$

This system has as solution $(x^*, y^*) = (0, 0)$ and its jacobian matrix is given by the following matrix

$$\begin{pmatrix} 2 - 2\frac{x}{\epsilon} & 1 - \frac{y}{\epsilon} \\ 1 & 1 \end{pmatrix}$$

On the other hand, the matrix $[x, \tilde{x}; F]$, with $x = (x_1, y_1)$ and $\tilde{x} = (x_2, y_2)$, is given by

$$\begin{pmatrix} 2 - \frac{x_1 + x_2}{\epsilon} & 1 - \frac{y_1 + y_2}{\epsilon} \\ 1 & 1 \end{pmatrix}$$

The parameter ϵ (when $\epsilon \to 0$) increases the condition number of the difference divided matrix for a given initial guess and the secant method should have problems of convergence to the solution for small values of ϵ. Moreover, notice that $F'(\epsilon, \epsilon)$ is not invertible.

We compute the maximum of the condition numbers of the linear systems that appear in the application of the secant methods ($||A|| \cdot ||A^{-1}||$) and the maximum of the conditions numbers in all the matrix multiplications in method (3) ($\frac{||A|| \cdot ||B||}{||AB||}$).

In Table 1, the vector (ϵ, ϵ) is inside of the ball containing the initial guess and the solution. The secant method diverges.

This numerical behavior is similar for smaller parameters of ϵ, as we can see in the Table 2 ($\epsilon = 10^{-3}$). The maximum of the condition numbers for the secant method is $8.34\ 10^2$ (too big) and for the method (3) smaller than 10. For this example,

Table 1
System (4). Errors for $(x_{-1}, y_{-1}) = (-2, 2), (x_0, y_0) = (-1, 1), ||B_0 F'(-2, 2)|| \leqslant 10^{-3}$ and $\epsilon = 1$.

n	Secant	Method (3)
1	$1.41\ 10^{0}$	$1.21\ 10^{0}$
2	$2.12\ 10^{0}$	$4.77\ 10^{-1}$
3	$4.77\ 10^{0}$	$1.46\ 10^{-1}$
4	$2.41\ 10^{1}$	$2.65\ 10^{-2}$
5	$6.19\ 10^{2}$	$1.56\ 10^{-3}$
6	$4.07\ 10^{5}$	$8.71\ 10^{-6}$
7	$1.75\ 10^{11}$	$3.68\ 10^{-10}$
8	$3.97\ 10^{22}$	$3.44\ 10^{-19}$

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