Applied Mathematics and Computation 247 (2014) 996–1002

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Dynamical analysis of a logistic equation with spatio-temporal delay

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ARTICLE INFO

Keywords:
Logistic equation
Spatio-temporal delay
Stability
Traveling wave front

ABSTRACT

This paper is concerned with a logistic equation with spatio-temporal delay. The local asymptotic behavior of positive constant steady-state solution of the equation is considered. In particular, by using the iterative technique, sufficient conditions are established for the existence of traveling wave front solution connecting the zero and the positive constant steady-state solution. Finally, numerical simulations supporting the theoretical analysis are also included.

1. Introduction

In recent years, many mathematical models, involving reaction–diffusion equations with spatially and temporally nonlocal terms in the form of the convolution of a kernel with the dependable variable, have attracted much attention in population biology since they are believed to be more realistic than the usual kind of reaction–diffusion models for certain population dynamics. For more biological background and derivations of such models, see [1–4,6,7,5,8–10].

In this paper, we consider the following diffusive logistic equation with spatio-temporal delay

$$\frac{\partial u(x,t)}{\partial t} = \triangle u(x,t) + u(x,t)[1-(G*u)(x,t)], \quad x\in\Omega,\ t>0, \tag{1.1}$$

where $u(x,t)$ is the population density at location x and time t. The convolution $G*u$ is defined by

$$(G*u)(x,t)=\int_{-\infty}^{t}\int_{\Omega} f(x,y,t-s)k(t-s)u(y,s)dyds.$$

The nonlocal growth rate per capita in (1.1) incorporates the possible dispersal of the individuals during the maturation period, hence it is a more realistic model. We assume that it takes the form of the so-called weak kernel [11] as follows $k(s)=\frac{1}{\tau}e^{-\frac{s}{\tau}}, \tau>0$, where τ is the average delay. f is a weighting function describing the distribution at past time of the individuals of the species u who is in position x at time t. For more details on the choice of kernel functions and the background of spatial–temporal delay, see [1–5,8].

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[1] Supported by the National Natural Science Foundation of China (11226153, 11301147), the Science and Technology Research Foundation Awarded by Educational Commission of Henan Province of China (14A110021) and the Fundamental Research Fund of Henan University (2012YBZR032).
[2] Supported by the National Natural Science Foundation of China (11201359) and the Fundamental Research Funds for the Central Universities.

http://dx.doi.org/10.1016/j.amc.2014.09.067
0096-3003/

It is well-known that the prototypical delayed reaction–diffusion equation is the diffusive logistic equation following the pioneering work of Hutchinson [12]

$$\frac{\partial u(x,t)}{\partial t} = \triangle u(x,t) + u(x,t)[1 - u(x,t-\tau)]. \tag{1.2}$$

For a long time, it has been recognized that delays not only can cause the loss of stability but also induce various oscillations and periodic solutions, see [13–16]. For example, for the Neumann boundary value problem, (1.2) has been considered in [17,18] and they considered the stability and related Hopf bifurcation from the homogeneous equilibrium. Busenberg and Huang [19] studied the Hopf bifurcation of (1.2) and Dirichlet boundary condition proposed by Green and Stech [20]. For more the conclusions of (1.2), see [22,21]. They showed that the unique spatially positive steady-state solution loses the stability for a large delay and a Hopf bifurcation occurs so that the system exhibits oscillatory pattern. However, the local asymptotic stability of the positive constant steady-state solution of (1.1) is found in this work. In addition, the traveling wave solutions for an equation in form of (1.2) have been considered in many papers [2,3,24–26,23]. In this paper, by using an iterative technique recently developed by Wang, Li and Ruan [8], sufficient conditions are established for the existence of traveling wave front solution connecting the zero and the positive equilibria in reaction–diffusion equation with spatio-temporal delay (1.1).

The rest of this paper is organized as follows. In Section 2, the local asymptotic behavior of positive constant steady-state solution of (1.1) is considered. In Section 3, the existence of traveling wave front is demonstrated. Finally, numerical simulations supporting the theoretical analysis are also included.

2. Local stability

In this section, we focus on the local asymptotic behavior of positive constant steady-state solution of (1.1). We consider Eq. (1.1) with the following boundary value conditions

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega \tag{2.1}$$

and initial value conditions

$$u(x,\theta) = \phi(x,\theta) > 0, \quad x \in \Omega, \theta \in (-\infty, 0]$$

and $\int_0^\infty k(s)ds = 1$.

$$f(x,y,t) = \frac{1}{\pi} + \frac{2}{\pi}\sum_{n=1}^{\infty} e^{-n^2 t}\cos nx \sin ny$$

is a fundamental solution of the heat equation

$$\begin{cases} \frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial y^2}, & y \in \Omega,\ t > 0, \\ \frac{\partial f}{\partial y} = 0, & y \in \partial\Omega,\ t > 0, \\ f(x,y,0) = \delta(x-y), \end{cases} \tag{2.2}$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$; ν is the unit outward normal vector on the boundary of Ω and the Neumann boundary conditions imply that the species have zero flux across the domain boundary $\partial\Omega$.

Let $v(x,t) = \int_{-\infty}^t \int_\Omega f(x,y,t-s)k(t-s)u(y,s)dyds$, then (1.1) can be rewritten as the following system

$$\begin{aligned} &\frac{\partial u(x,t)}{\partial t} = \triangle u(x,t) + u(x,t)[1 - v(x,t)], \quad x \in \Omega,\ t > 0, \\ &\frac{\partial v(x,t)}{\partial t} = \triangle v(x,t) + \frac{1}{\tau}[u(x,t) - v(x,t)], \quad x \in \Omega,\ t > 0, \\ &\frac{\partial u(x,t)}{\partial \nu} = 0, \quad \frac{\partial v(x,t)}{\partial \nu} = 0, \quad x \in \partial\Omega,\ t > 0, \\ &u(x,\theta) = \phi(x,\theta) > 0, \quad v(x,\theta) = \psi(x,\theta) > 0, \quad x \in \overline{\Omega},\ \theta \in (-\infty, 0]. \end{aligned} \tag{2.3}$$

It is well known that the linear operator Δ on Ω with homogeneous Neumann boundary conditions has the eigenvalues $-\mu_i(\mu_i \geqslant 0, i = 0,1,2\cdots)$. The characteristic equation for the linearized system (2.3) on positive steady-state solution $u^* = 1$ is

$$\lambda^2 + \left(2\mu_i + \frac{1}{\tau}\right)\lambda + \mu_i^2 + \frac{1}{\tau}\mu_i + \frac{1}{\tau} = 0. \tag{2.4}$$

It is obvious that Eq. (2.4) has no zero roots since $2\mu_i + \frac{1}{\tau} > 0, \mu_i^2 + \frac{1}{\tau}\mu_i + \frac{1}{\tau} > 0$. Consequently, all roots of Eqs. (2.4) have negative real parts. Therefore, we have the following result: