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On the iterative convergence of harmony search algorithm and a proposed modification

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ABSTRACT

Inspired by the improvisation process of music players, a population-based meta-heuristic algorithm-harmony search (HS) has been proposed recently. HS is good at exploitation, but it can be poor at exploration, and its convergence performance can also be an issue in some cases. To address these disadvantages, the distance bandwidth (*bw*) adjusting methods proposed in recent literatures are summarized and the exploration ability of HS improvisation is investigated in this paper. Further, the relationship between improvisation exploration and each parameter under asymmetric interval is derived, and an iterative convergence sufficiency of the iteration equation which consists of variance expectation and mean expectation is proven theoretically. Based on these analyses, a modified harmony search (MHS) algorithm is proposed. Moreover, the effects of the key parameters including HMS, PAR and HMCR on the performance of the MHS algorithm are discussed in depth. Experimental results reveal that the proposed MHS algorithm performs better than HS as well as its state-of-the-art variants and other classic excellent meta-heuristic approaches.

1. Introduction

A musician composes a dulcet and harmonious musical chord by means of personal music experience and note adjusting. This phenomenon is analogous to a meta-heuristic swarm optimization algorithm to find a global optimal solution or a near optimal solution via searching experience and effective exploration. Inspired by this observation, Geem et al. developed a new meta-heuristic algorithm, i.e. harmony search (HS) algorithm [1], in 2001.

HS not only has the merits of common meta-heuristic algorithm but it also has its own characteristics. Several characteristics of HS are summarized in [2–4]: (a) HS algorithm combines all of the existing harmony vectors to produce a new harmony vector, whereas a genetic algorithm (GA) only uses the two parent vectors, particle swarm optimization (PSO) only considers the individual best position and the global best position of the whole swarm, and differential evolution algorithm (DE) only integrates three different individuals. (b) The whole process of improvisation is completely random. Thus a HS can regard as a global random searching model. For memory consideration, pitch adjustment or random selection, regardless what rules the HS implemented, the precondition is the random number must satisfy the value of the control parameters. (c) HS independently adjusts each variable via improvisation, unlike PSO handles a solution vector by unitary rule. Over

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the last decades, HS has been successful applied to a wide variety of optimization problems, such as environmental/economic dispatch [5], groundwater management model [6], non-technical losses detection [7], hybrid stable adaptive fuzzy controller design [8], medium-term sales forecasting in fashion retail supply chains [9]. Especially in the recent years, the HS algorithm likes other heuristic methods such as genetic algorithm, particle swarm algorithm and differential evolution algorithm has attracted much attention and has been applied to various science and engineering problems [26–28].

Though HS has many obvious advantages, it also suffers from a serious problem as other meta-heuristics such as PSO [10] and DE [11] do. That is, it can be trapped in performing local search for solving numerical optimization problems [2]. Moreover, its optimization performance is quite sensitive to key control parameters. Therefore, how to effectively fine-tune the key control parameters (especially *bw*) in the process of improvisation is a key research focus in harmony search algorithm. An unreasonable *bw* adjustment strategy can cause HS blind search, and thus reduce the optimizing effectiveness and precision of a HS. So many attempts have been pursued to overcome this disadvantage via adjusting the value of *bw*. A number of contributions about the adjustment of parameter *bw* are summarized as follow. Mahdavi et al. proposed an improved harmony search algorithm (IHS) [2] for solving various standard engineering optimization problems. IHS can enhance the fine-tuning characteristic and convergence rate of harmony search through dynamically fine-tune the values of parameters PAR and *bw* with generations, in which *bw* is exponentially decreased with generations. Gao et al. presented an adaptive harmony search-particle swarm optimization (AHSPSO) algorithm [12] that the values of PAR and *bw* are adaptively updated according to the change of variables in the current harmony memory, where the value of *bw* is automatically controlled by the values of the current maximum and minimum variables and the upper and lower bounds. A new variant of harmony search, namely self-adaptive harmony search (SAHS), developed by Wang et al. [13]. SAHS applies the consciousness to automatically adjust the value of the control parameters. Specifically, the value of *bw* is automatically generated according to the maximum and minimum values of the variables in the current harmony memory (HM). Inspired by the conception of swarm intelligence, global best harmony search (GHS) algorithm was proposed by Mahamed et al. [3]. The best harmony replaced the pitch adjustment in GHS. GHS discarded the parameter *bw*. Majid Jaberipour et al. designed two improved harmony search algorithms: proposed harmony search (PHS) and improving proposed harmony search (IPHS) [14]., the main difference between the two approaches and the original HS is the PHS and IPHS employ approximate derivatives to adjust the value of *bw*.

Although a significant amount of research has already been undertaken to ameliorate the optimization performance of HS by regulating its key parameters and/or integrating it with other useful evolution strategies like chaotic maps [15], multi-population [16] or dynamic sub-swarms [17], neighborhood search [18] or optimization techniques such as particle swarm optimization [19], differential evolution [20–21], colony strategy [22], to the best of our knowledge, there is not much work devoted to analyze the mathematical theory of the underlying search mechanisms of HS. Das et al. analyzes the evolution of the population variance of HS and discusses the relationship between the explorative power of HS and the control parameters, and then a modified harmony search named EHS was proposed [23]. In EHS, the parameter *bw* is dynamically updated according to the standard deviation of the current harmony, which is aimed at improving the exploratory power of the harmony search algorithm. EHS is good at exploration but poor at exploitation. Here exploration means that an algorithm can independently seek for the global optimum solution in the new regions of a large search space, while exploitation means that an algorithm can apply the existing information to find better solutions. In addition, its search precision and convergence speed are also an issue in some cases. A reasonable balance between exploration and exploitation are beneficial to the performance of an algorithm [23–25]. Because many modified HS still cannot escape local minimum and adjust algorithm parameters effectively, so the relationship between the search mechanism of HS and the parameters is a very significant area for future research. That deserves a lot more attention in the future and this paper is thus motivated to focus on this research.

The main contributions of this paper are as follows: (i) based on the previous work effort of Das et al., the mathematical theory of the underlying search mechanisms of HS is further analyzed and discussed with the view of balancing exploration and exploitation. The relationship between improvisation exploration and each parameter under asymmetric interval is derived. (ii) The iteration equation about variance expectation and mean expectation is first established. Then an iterative convergence sufficiency condition is proven theoretically. (iii) Based on the analyses, a simple effective modification of the original harmony search algorithm is proposed for solving numerical optimization problems. Our experimental results reveal that the proposed MHS algorithm performs better than HS and the other state-of-the-art HS variants and the other excellent meta-heuristic approaches.

The remainder of the paper is organized as follows. The original HS algorithm and its modified state-of-the-art HS variants are reviewed in Section 2. Section 3 summarizes comprehensively the mathematical analysis of the HS searching mechanism. Section 4 further analyzes the search mechanism of HS under general interval in detail, and constructs an iteration equation to research iterative convergence sufficiency condition by balancing exploration and exploitation. A simple modified HS is proposed in Section 5. Section 6 presents a large number of experiments to investigate the effectiveness and robustness of the proposed algorithm, and makes a comparison with the other state-of-the-art meta-heuristic algorithms. Finally, a summary of this paper is given in Section 7.

2. Harmony search algorithm and parameter design of its variants

In this section, we provide an overview on harmony search algorithm and parameter design of its variants.