Applied Mathematics and Computation 234 (2014) 36–51

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Discrete singular convolution element method for static, buckling and free vibration analysis of beam structures

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ARTICLE INFO

Keywords:
Discrete singular convolution element method
Taylor's series expansion
Stepped beam
Discontinuities

ABSTRACT

A new version of discrete singular convolution (DSC) algorithm, called the DSC element method, is proposed. The new method combines the high computational efficiency and accuracy of the original DSC algorithm with the convenience and generality of the finite element method in application to structural analysis. The shortage existing in the original DSC algorithm has been overcome. Besides, the proposed method provides a unique way to apply various boundary conditions in using DSC algorithm. Two types of DSC beam element are presented and a variety of problems, such as static, buckling and free vibrations with discontinuities in geometry or load, have been successfully solved by the proposed method. Numerical results show that the proposed DSC element method is an accurate and efficient method for analyzing beam structures.

1. Introduction

In past a few years, the discrete singular convolution (DSC) algorithm, pioneered by Wei in 1999 [1], has been well developed. It has been demonstrated that the DSC combines the high accuracy of a global method with the flexibility of a local method for science and engineering applications [2–4]. Some challenge problems have been solved by using this method successfully, such as numerical solutions to the Fokker–Planck equation [1], Schrodinger equation [5], Navier–Stokes equation [6] and higher-order modes vibration in the structural analysis [7–9].

With much effort spent on the development of the DSC algorithm, the DSC becomes a potential and promising numerical method. Due to its effectiveness and reliability, the DSC has been extensively explored for a variety of applications, such as, vibration analysis of plates with irregular internal supports [10], conical panels [11] and three-dimensional thick plate problems [12], free vibration of laminated composite thin plates [13] and shells [14,15], arbitrary straight-sided quadrilateral plates [16], Mindlin plates with mixed edge supports [17], elastic wave propagations in one-dimensional structures [18], nonlinear analysis of thin rectangular plates on Winkler–Pasternak elastic foundations [19], and non-linear buckling analysis [20].

Due to introduce M (the half computational bandwidth of the DSC method) fictitious points (FPs) outside each boundary, some difficulties exist in the applications of the DSC to structural analysis, e.g., the accurate treatment of free boundary conditions for anisotropic rectangular thin plates, and for beams and plates with discontinuities in geometry or load.

The methods of symmetric extension can be used efficiently for applying the clamped boundary conditions and the method of anti-symmetric extension can accurately treat the simply supported boundary conditions for isotropic and orthotropic

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http://dx.doi.org/10.1016/j.amc.2014.01.165
0096-3003/

Nomenclature

A	cross-sectional area
A_{ik}	weighting coefficients of the first order derivative
B_{ik}	weighting coefficients of the second order derivative
C_{ik}	weighting coefficients of the third order derivative
D_{ik}	weighting coefficients of the fourth order derivative
E	Young's modulus
I	moment of inertia
L	beam length
$2M+1$	the computational bandwidth
M^*	bending moment
N	total number of grid points
Q^*	shear force
ρ	density of plate material
ω	the circular frequency
Superscripts $-$	the limiting value of a function from left side of a step
Superscripts $+$	the limiting value of a function from right side of a step
ADM	adomian decomposition method
C	clamped edge
CEM	composite element method
DQM	differential quadrature method
DQEM	differential quadrature element method
DSC	discrete singular convolution
DSC-LK	DSC with the non-regularized Lagrange delta sequence kernel
DSC-RSK	DSC with regularized Shannon's delta kernel
DSC-S	DSC with symmetric extension method
F	free edge
FEM	finite element method
FPs	fictitious points
DOF	degree of freedom
MIB	matched interface and boundary method
S	simply supported edge
$L_{M,k}(x)$	Lagrange interpolation function
$P(x)$	axial compressive force
$q(x)$	distributed load
$w_{(n)}(x)$	the nth order derivative of $w(x)$
$w(x)$	deflection
$\delta^{(n)}_{\sigma,\Delta}(x-x_k)$	a collective symbol for the delta kernels of Dirichlet type

rectangular thin plates [10]. For applying the free boundary conditions, Wei and his co-workers proposed the matched interface and boundary (MIB) method [21,22]. Although the method works well for beams with free edges and accurate frequencies are obtained, however, the accuracy of frequencies for isotropic rectangular thin plates with free edges is not as good as expected. Later, the senior author proposed a general method, called Taylor's series expansion method [23–25], to apply various boundary conditions. Accurate frequencies for beams with free ends, isotropic or orthotropic rectangular thin plates with free edges [24,25], and anisotropic thin plates with simply supported boundary conditions [24] are obtained. However, the frequency accuracy for anisotropic rectangular thin plates with free edges is not as good as the one for orthotropic rectangular thin plates with free edges [25]. Therefore, the problem has not been solved completely yet and deserved further investigations.

Beams and plates with geometrical discontinuities are common in engineering applications. Analysis of stepped beams and plates has attracted many researchers' attentions due to their unique functions. Analytical methods are complicated [26–30], only applicable for a few cases, and often fails [27], thus numerical simulation is resorted to solve the problems for engineering applications. Various methods have been used, such as, the composite element method (CEM) [31], the adomian decomposition method (ADM) [32,33], and the differential quadrature element method (DQEM) [34]. However, the above mentioned and most common numerical methods cannot yield accurate higher order mode frequencies. Although the DSC algorithm has been demonstrated to be a viable option to handle the challenging problem of vibrating at higher-order