



# Single machine group scheduling with decreasing time-dependent processing times subject to release dates



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## ABSTRACT

In this paper we investigate a single machine scheduling problem with decreasing time-dependent processing times and group technology assumption. By the decreasing time-dependent processing times and group technology assumption, we mean that the group setup times and job processing times are both decreasing linear functions of their starting times. We want to minimize the makespan subject to release dates. We show that the problem can be solved in polynomial time.

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## 1. Introduction

Traditional scheduling models and problems usually involve jobs with constant, independent processing times. In practice, however, we often encounter settings in which job processing times increase or decrease over time, e.g., in the modeling of the forging process in steel plants, finance management and scheduling maintenance or learning activities. This is why in recent years more and more researchers are considered scheduling problems with **time-dependent processing times**. Extensive surveys of different scheduling models and problems involving jobs with start time dependent processing times can be found in Alidaee and Womer [1], Cheng et al. [2] and Gawiejnowicz [3]. More recent papers which have considered scheduling problems with job time-dependent processing times include Lee et al. [4], Wu et al. [5], Wu and Lee [6], Wang et al. [7], Wang et al. [8], Lee et al. [9], He et al. [10], Chung et al. [11], Li et al. [12], Yang and Kuo [13], Wang and Sun [14], Wei and Wang [15], Yang and Yang [16], Yang et al. [17], Zhang and Yan [18], Zhao and Tang [19], Lee et al. [20], Zhu et al. [21], Huang and Wang [22], Zhao and Tang [23], Wang et al. [24], Yang and Wang [25], Sun et al. [26], Wang et al. [27], Wang et al. [28], Lee and Lu [29], Zhang and Luo [30], Liu et al. [31], Wu et al. [32], Wang and Wang [33–37], Xu et al. [38].

Generally, there are two types of models describing this kind of scheduling. The first type is devoted to the problems in which the processing time of a job is an increasing (non-decreasing) function of its starting time (deteriorating job processing times). The second type concerns problems in which the processing time of a job is a decreasing (non-increasing) function of its starting time (shortening processing times). Wang et al. [24] considered flow shop scheduling problem with decreasing linear deterioration, i.e., the (actual) processing time of job  $J_j$  on machine  $M_h$  is  $p_{hj} = a_{hj}(1 - bt)$ ,  $h = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, n$ , where  $a_{hj}$  is the normal (basic) processing time of job  $J_j$  on machine  $M_h$ ,  $t$  is its starting time,  $b > 0$  is a decreasing rate such that  $(1 - bt) > 0$ . When some dominance relations between  $m - 1$  machines can be satisfied, they showed that the makespan minimization problem can be solved in polynomial time. Wang et al. [27] considered single

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machine scheduling problem with decreasing linear deterioration, i.e., the (actual) processing time of job  $J_j$  is  $p_j = a_j(a - bt)$ ,  $j = 1, 2, \dots, n$ , where  $a > 0$ ,  $b > 0$  and  $(a - bt) > 0$ . For the total absolute differences in waiting times minimization problem, they proved several properties of an optimal schedule, and proposed two heuristic algorithms.

On the other hand, scheduling models and problems in a group technology (GT) environment have attracted numerous researchers due to their frequent real-life occurrence (Potts and Van Wassenhove [39], Webster and Baker [40], Liaee and Emmons [41], Janiak et al. [42], Bozorgirad and Logendran [43], Ji et al. [44]). Group technology that groups similar products into families helps increase the efficiency of operations and decrease the requirement of facilities.

It is natural to study the situations where group scheduling and time dependent processing times are combined. To the best of our knowledge, only a few results concerning scheduling problems with time dependent processing times and group technology simultaneously are known. Wu et al. [5] considered a situation where group setup times and job processing times are both described by a simple linear deterioration function, i.e., the (actual) processing time of job  $J_j$  in group  $G_i$  is  $p_{ij} = \beta_{ij}t$ ,  $i = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, n_i$ , where  $\beta_{ij} > 0$  is the deterioration rate of job  $J_j$  in group  $G_i$ ; the (actual) setup time of group  $G_i$  is  $s_i = \eta_i t$ , where  $\eta_i > 0$  is the deterioration rate of the setup time for group  $G_i$ . Using the extended three-field notation scheme (Graham et al. [45]), they proved that the makespan minimization problem ( $1|p_{ij} = \beta_{ij}t, s_i = \eta_i t, GT|C_{\max}$ ) and the total completion time minimization problem ( $1|p_{ij} = \beta_{ij}t, s_i = \eta_i t, GT|\sum \sum C_{ij}$ ) can be solved in polynomial time, where  $C_{ij}$  represents the completion time of job  $J_j$  in group  $G_i$ ,  $C_{\max} = \max\{C_{ij} | i = 1, 2, \dots, m; j = 1, 2, \dots, n_i\}$  represent makespan of a given schedule. Wu and Lee [6] considered a situation where group setup times and job processing times are both described by a linear deterioration function, i.e.,  $p_{ij} = \alpha_{ij} + bt$ ,  $i = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, n_i$ , and  $s_i = \delta_i + gt$ ,  $i = 1, 2, \dots, m$ , where  $\alpha_{ij} \geq 0$  is the normal (basic) processing time of job  $J_j$  in group  $G_i$ ,  $b > 0$  is the deterioration rate of jobs,  $\delta_i \geq 0$  is the normal (basic) setup time for group  $G_i$ ,  $g$  is a deterioration rate of setup times. They showed that the makespan minimization problem ( $1|p_{ij} = \alpha_{ij} + bt, s_i = \delta_i + gt, GT|C_{\max}$ ) remain polynomially solvable. For the sum of completion times problem, they showed that the problem remains polynomially solvable under the assumption that the numbers of jobs in each group are equal. Wang et al. [7] considered the following model:  $p_{ij} = \alpha_{ij}(a + bt)$ ,  $i = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, n_i$ , and  $s_i = \delta_i(a + bt)$ ,  $i = 1, 2, \dots, m$ . They proved that the problems  $1|p_{ij} = \alpha_{ij}(a + bt), s_i = \delta_i(a + bt), GT|C_{\max}$  and  $1|p_{ij} = \alpha_{ij}(a + bt), s_i = \delta_i(a + bt), GT|\sum \sum w_{ij}C_{ij}$  can be solved in polynomial time, where  $w_{ij}$  denote the weight of job  $J_j$  in group  $G_i$ . Wang et al. [8] considered a situation where group setup times and job processing times are both described by a general linear deterioration function, i.e.,  $p_{ij} = \alpha_{ij} + \beta_{ij}t$ ,  $i = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, n_i$ , and  $s_i = \delta_i + \eta_i t$ ,  $i = 1, 2, \dots, m$ . They proved that the makespan minimization problem ( $1|p_{ij} = \alpha_{ij} + \beta_{ij}t, s_i = \delta_i + \eta_i t, GT|C_{\max}$ ) can be solved in polynomial time. Wei and Wang [15] proved that the problems  $1|p_{ij} = \beta_{ij}t, s_i = \eta_i t, GT|\sum w_{ij}C_{ij}^2$  and  $1|p_{ij} = \beta_{ij}t, s_i = \eta_i t, GT|\sum w_{ij}W_{ij}^2$  can be solved in polynomial time, where  $W_{ij} = C_{ij} - p_{ij}$  is the waiting time of job  $J_j$  in group  $G_i$ . Yang and Yang [16] considered scheduling problems under the effects of deterioration and learning under group technology, i.e.,  $p_{ij} = \alpha_{ij}r^{a_i}$ ,  $p_{ij} = \alpha_{ij}(1 + \sum_{q=1}^{r-1} \alpha_{ijq})^{a_i}$ ,  $s_i = \delta_i t$ , where  $a_i \leq 0$  denote the learning factor of group  $G_i$ , and  $r$  denote the job position. They showed that the makespan minimization problems can be solved in polynomial time. They also showed that the total completion time minimization problem have a polynomial optimal solution under agreeable restrictions. Zhang and Yan [18] considered group scheduling with deterioration and learning effect on a single machine, i.e.,  $p_{ij} = (\alpha_{ij} + bt)r^{a_1}k^{a_2}$ ,  $s_i = \delta_i r^{a_1}$ , where  $a_1 \leq 0$  and  $a_2 \leq 0$  denote the learning effect, and  $r$  and  $k$  denote the group position and the job position. They showed that the makespan and the total completion time minimization problems can be solved in polynomial time. They also showed that the maximum lateness minimization problem have a polynomial optimal solution under agreeable conditions. Lee and Lu [29] considered the problem  $1|p_{ij} = \beta_{ij}t, s_i = \eta_i t, GT|\sum w_{ij}U_j$ , where  $U_j(\pi) = 1$  if  $C_j(\pi) > d_j$  and 0 otherwise, where  $d_j$  denote the due date of job  $J_j$ , they proposed a branch-and-bound algorithm to solve this problem.

Wang and Sun [14] considered the group scheduling with linearly decreasing time-dependent setup times and job processing times on a single machine, i.e.,  $p_{ij} = \alpha_{ij} - \beta_{ij}t$ ,  $s_i = \delta_i - \eta_i t$ , where  $\beta_{ij}$  is the decreasing rate of job  $J_j$  in group  $G_i$ . They proved that the problem  $1|p_{ij} = \alpha_{ij} - \beta_{ij}t, s_i = \delta_i - \eta_i t, GT|C_{\max}$  can be solved in polynomial time. For a special case  $\beta_{ij} = b\alpha_{ij}$ ,  $\eta_i = b\delta_i$ , they proved that the problem  $1|p_{ij} = \alpha_{ij}(1 - bt), s_i = \delta_i(1 - bt), GT|\sum w_{ij}C_{ij}$  can be solved in polynomial time.

Wang et al. [28] considered scheduling with independent setup times, ready times, and deteriorating job processing times under group technology assumption on a single machine. They proved that the problem  $1|r_{ij}, p_{ij} = \beta_{ij}t, s_i, GT|C_{\max}$  have a polynomial optimal solution under an agreeable condition, where  $r_{ij}$  denote the ready times (release dates) of job  $J_j$  in group  $G_i$ . Xu et al. [38] considered the problem  $1|r_{ij}, p_{ij} = \beta_{ij}(a + bt), s_i, GT|C_{\max}$ . For some special cases, they proved that the problem can be solved in polynomial time.

In this paper we consider single machine group scheduling with release dates, decreasing time-dependent setup times and job processing times (to our knowledge for the first time) at the same time. We show that the makespan minimization problem can be solved in polynomial time. The remaining part of the paper is organized as follows. In the next section, a precise formulation of the problem is given. The problem of minimizing the makespan is given in the Section 3. The last section contains some conclusions.

## 2. Problem formulation

The single machine group scheduling problem with group setup times can be stated as follows: There are  $n$  jobs grouped into  $m$  groups, and these  $n$  jobs are to be processed on a single machine. A setup time is required if the machine switches

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