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Positivity-preserving rational bi-cubic spline interpolation for 3D positive data

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ABSTRACT

This paper deals with the shape preserving interpolation problem for visualization of 3D positive data. A required display of 3D data looks smooth and pleasant. A rational bi-cubic function involving six shape parameters is presented for this objective which is an extension of piecewise rational function in the form of cubic/quadratic involving three shape parameters. Simple data dependent constraints for shape parameters are derived to conserve the inherited shape feature (positivity) of 3D data. Remaining shape parameters are left free for designer to modify the shape of positive surface as per industrial needs. The interpolant is not only local, C^1 but also it is a computationally economical in comparison with existing schemes. Several numerical examples are supplied to support the worth of proposed interpolant.

1. Introduction

Shape preserving interpolation problem for visualization of 3D positive data is one of the basic problem in computer graphics, computer aided geometric design, data visualization and engineering. It also arises frequently in many fields including military, education, art, medicine, advertising, transport, etc. Curve and surface design plays a significant role not only in these fields but also in manufacturing different products such as ship design, car modeling and airplane fuselages and wings.

In many interpolation problems, it is essential that the interpolant conserves some inherited shape features of data like positivity, monotonicity and convexity. The goal of this paper is to conserve the hereditary characteristic (positivity) of 3D data. Positivity-preserving problem occurs in visualizing a physical quantity that cannot be negative which may arise if the data is taken from some scientific, social or business environments. Depreciation of the price of computers in the market is an important example of positive data. Ordinary spline methods usually ignore these characteristics thus exhibiting undesirable inflections or oscillations in resulting curves and surfaces. Due to this reason, many investigations during the past years have been directed towards shape preserving interpolation schemes which are quoted as: A rational cubic [1], bi-cubic interpolants [2,7–8] and rational bi-cubic partially blended function [10–12] have a common feature in a way that no extra knots are used for shape preservation of positive 2D and 3D data. In contrast, the piecewise cubic Hermite interpolation [3–4] and piecewise bi-cubic function [5] conserved the shape of data by inserting one or two extra knots in the interval where the interpolants do not conserve the desired shape characteristics of data. Shape preserving interpolants problem

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for visualization of positive data has been solved by using C^1 piecewise rational bi-cubic functions [2,7–8], rational bi-quadratic splines [9] and rational bi-cubic partially blended functions [10–13] with shape parameters. Simple data dependent conditions were derived for shape parameters to attain the desired shape feature of 3D data using rational splines [2,7–13].

In this paper, a C^1 piecewise rational bi-cubic spline scheme with six shape parameters is developed to handle the problem of constructing a positivity-preserving surface through 3D positive data. This method is a contribution towards the advancement of such results that have been carried out by many authors. The method has many outstanding features like:

1. Abbas et al [2] extended the rational cubic function to rational bi-cubic function (cubic/cubic) in order to conserve a positive surface. This function was proved to be successful to interpolate the positive data for only non-zero partial derivatives in contrast the proposed scheme works for any value of partial derivatives.
2. No need of extra knots in the proposed interpolant. In contrast, the piecewise bi-cubic interpolant [5] achieves the required shape of surface by inserting of extra knots in the subinterval where the interpolant loses positivity of surface.
3. The schemes [7,10] do not allow the designer to refine the positive surface as per consumer's demand. Whereas, this job is done by introducing free parameters which they are used in the description of rational bi-cubic function (5).
4. Hussain [8] constructed a rational bi-cubic function with eight shape parameters in cubic/cubic form. Data dependent constraints were derived for four shape parameters to conserve the positivity. Whereas, the proposed scheme is computationally economical as compared to scheme [8] because it has bunch of shape parameters in every rectangular patch.
5. The proposed positive interpolant has been demonstrated through several numerical examples and it is found not only local but also produces graphical pleasant results as compared to existing schemes [2,7–12] due to less number of constraints for shape parameters and flexibility bestowed to designer for refinement of surfaces as per consumer's demand.
6. In [9], the smoothness of bi-quadratic interpolant is C^0 while in this paper it is C^1.
7. In [10], the authors claimed that the rational bi-cubic partially blended functions (coon patches) generated a positive surface but unfortunately the visual models did not depict the positive surfaces due to the coon patches because they conserved the shape of data only on the boundaries of patch not inside the patch. In contrast, the proposed rational bi-cubic interpolant conserves the shape of data everywhere in the domain.
8. The proposed surface scheme is unique in its representation and it works well for both uniform and non-uniform space data. The proposed scheme is equally applicable for the data with derivative or without derivatives while the scheme developed by Casciola et al [13] work if partial derivatives at the knots are known.

This paper is organized as follows: A review of rational cubic spline function [1] with three shape parameters with shape control analysis is discussed in Section 2. The extension of rational cubic function to a rational bi-cubic function for the interpolation of regular 3D positive data is presented in Section 3. The arithmetic mean method for derivative approximation is discussed in the Section 4. Positivity-preserving interpolating rational bi-cubic scheme is constructed in Section 5. Several numerical examples are given in Section 6 to prove the worth of scheme. The concluding remarks are presented to end the paper.

2. Rational cubic spline function

Rational spline models has more authority than polynomial spline models as it can accommodate a much wider range of shapes, moderately simple form, better interpolatory properties, excellent extrapolatory powers, typically smoother, easy to handle computationally, less oscillatory and to model complicated structure with a fairly low degree in both the numerator and denominator. In this section, we rewrite the rational cubic spline function developed by Abbas et al. [1].

Let $\{(x_i, f_i), i = 0, 1, 2, \ldots, n\}$ be the given set of data points such that $x_i < x_{i+1}, i = 0, 1, 2, \ldots, n-1$. In each subinterval $I = [x_i, x_{i+1}], i = 0, 1, 2, \ldots, n-1$, a piecewise rational cubic function is defined as:

$$S(x) \equiv S_i(x) = \frac{\sum_{i=0}^{3}(1-t)^{3-i}t^i\xi_i}{q_i(t)}. \tag{1}$$

Let $S'(x)$ denotes the first ordered derivative with respect to x. The following conditions are imposed on rational cubic function (1) for the smoothness as:

$$\begin{aligned} &S(x_i) = f_i, \quad S(x_{i+1}) = f_{i+1}, \\ &S'(x_i) = d_i, \quad S'(x_{i+1}) = d_{i+1}. \end{aligned} \tag{2}$$

From (2), the values of unknown coefficients are

$$\begin{aligned} \xi_0 &= \alpha_i f_i \\ \xi_1 &= f_i(2\alpha_i + \beta_i + \gamma_i) + \alpha_i h_i d_i \\ \xi_2 &= f_{i+1}(\alpha_i + 2\beta_i + \gamma_i) - \beta_i h_i d_{i+1} \\ \xi_3 &= \beta_i f_{i+1} \\ q_i(t) &= \alpha_i(1-t)^2 + (\alpha_i + \beta_i + \gamma_i)t(1-t) + \beta_i t^2, \end{aligned} \tag{3}$$