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A nonlocal meshless solution for flexural vibrations of double-walled carbon nanotubes

Keivan Kiani

Department of Civil Engineering, K.N. Toosi University of Technology, Tehran, Iran

A R T I C L E   I N F O

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A B S T R A C T

The true understanding of free vibration of double-walled carbon nanotubes (DWCNTs) plays a vital role in optimal design and dynamic control of such nanostructures. This paper is aimed to examine free flexural vibration of lengthy DWCNTs with arbitrary boundary conditions in the framework of nonlocal elasticity theory. The DWCNTs are embedded in an elastic medium and are subjected to initially axial forces. Equivalent continuum structures associated with the innermost and outermost tubes of the DWCNT are considered. The transverse and rotational interactions of the DWCNT with the surrounding elastic medium are also taken into account. The generalized equations of motion of lengthy DWCNTs are established based on the nonlocal Rayleigh beam theory. Seeking an analytical solution to the developed equations, particularly in their general form, is a very problematic task. As an alternative solution, an efficient numerical scheme is proposed. The effects of slenderness ratio, small-scale parameter, lateral and rotational stiffness of the surrounding matrix, and initially axial force on the first five natural frequencies of DWCNTs under different boundary conditions are comprehensively scrutinized.

1. Introduction

The discovery of carbon nanotubes (CNTs) has opened up a new world in the field of nanotechnology. Since the past three decades, four major forms of carbon materials have come to the real world; those are C_{60} by Kroto et al. [1] in 1985, multi-walled carbon nanotube (MWCNT) by Iijima [2] in 1991, single-walled carbon nanotube (SWCNT) by Bethune et al. [3] in 1993, and carbon nanofiber. Subsequent studies revealed that CNTs integrate astonishing rigid and toughness properties, such as exceptionally high elastic properties, huge elastic strain, and fracture strain sustaining capability [4–8]. Beyond any exaggeration, CNTs are the strongest fibers recognized to date. The Young's modulus of a SWCNT is around 1 TPa, which is five times greater than steel, while its density is only 1200–1400 kg/m^3 [9]. It implies that the macro-scale structures made of CNTs will be extremely lighter and stronger than steel frames. Further studies reported that the frequency of flexural vibration of CNTs could reach the level of GHz [10,11] or even THz [12]. These evidences indicate that CNTs could be regarded as the most capable reinforcement materials for the next generation of high frequency engineered structures. Thereby, they have attracted the attention of both engineering and scientific communities of various disciplines during recent years. As a result, understanding the true mechanisms of their vibrations under various boundary conditions would be of great advantageous in optimal design of such nanostructures as well as future macro-scale structures made of CNTs.

Generally, free vibration of MWCNTs can be explored by using an atomistic-based approach, a classical continuum-based model, or via a nonlocal continuum-based model. In the two later models, numerical schemes or analytical solutions may be

E-mail addresses: k_kiani@kntu.ac.ir, keivankiani@yahoo.com

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implemented for solving the governing equations. The choice of the model in any circumstance entails a tradeoff in that. For example, atomistic-based models commonly lead to more accurate results. However, they are really time-consuming and labor intensive, which is not the case for the models based on the classical or even the nonlocal continuum mechanics. In the continuum-based models, the CNT is treated as a continuous material with specific geometry and elastic materials' properties. The classical continuum models which are used for flexural vibration of DWCNTs or MWCNTs are commonly based on the traditional engineering models such as beams [13–24] and shells [25–27]. The major drawback of such models is that the length of the C–C bond is not included in their formulations. This matter becomes highlighted particularly when the length of the nanostructure would be comparable with the length of the C–C bond or even when the wavelength of the propagated sound wave is fairly as small as the length of the C–C bonds. In order to conquer such a shortcoming of the classical models, other sophisticated continuum-based theories have been developed during the past century such as couple stress theory [28], modified couple stress theory [29,30], nonlocal continuum theory [31,32], and higher gradient continuum theory [33]. A simple version of the nonlocal continuum theory of Eringen [31,32] is the most popular one among the nanotechnologist since it could be readily applied to the classical governing equations. In this fairly novel theory, the lengths of interatomic bonds are incorporated into the equations of motion via a so-called small-scale parameter. For exploring flexural vibration of CNTs, an investigation by Duan et al. [34] revealed that the value of the small-scale parameter to use generally relies on the length-to-diameter ratios, mode shapes, and boundary conditions of the SWCNT. For each problem, the value of the small-scale parameter can be appropriately selected through justification of the predicted dispersion curves by the nonlocal model with those of an atomistic-based approach. Conversely, it seems that further research works are still needed to investigate the feasibility of application of other non-classical theories to free vibration of nanostructures. To date, nonlocal beam models [35–40] and nonlocal shells models [41,42] have been exploited for studying wave propagation within DWCNTs as well as flexural vibration of DWCNTs. In the present work, a nonlocal beam model is employed since only flexural vibration of DWCNTs is of particular interest. Among the beam theories, the Rayleigh beam theory is implemented since the understudy DWCNT is a lengthy nanostructure, and it is expected that the ratio of the shear strain energy to the flexural one would be negligible. Unlike the Euler–Bernoulli beam model, the rotary inertial of the DWCNT is also taken into account by the Rayleigh beam theory.

A brief survey of the literature displays that the research works on the vibration of DWCNTs in the context of nonlocal continuum theory of Eringen are limited to particular cases. For example, most of the previously published works were devoted to study free flexural vibration of simply supported DWCNTs [35,37,39] or flexural vibration of simply supported DWCNTs under external loads [43,44]. The main reason of this fact is that finding an analytical solution to the nonlocal governing equations pertinent to linear free flexural vibration of DWCNTs which are embedded in an elastic medium under arbitrary boundary conditions as well as initially axial forces is a very problematic task. To conquest such a dilemma, efficient numerical schemes could be implemented. In this work, reproducing kernel particle method (RKPM) is employed for solving the nonlocal equations of motion of lengthy DWCNTs. RKPM is an efficient meshless technique which is initially developed by Liu et al. [45]. To date, this method has been successfully applied to various engineering problems [46–49]. In contrast to finite element method (FEM), RKPM employs higher-order shape functions (i.e., interpolants) according to the used base function and window function. In FEM analysis, node-based meshes are used for discretizing of the spatial domain while in RKPM analysis, particles are used for this purpose. This matter provides RKPM for a wide class of continuum mechanics' problems, especially those undergoing mesh distortion, moving boundaries, higher gradients as well as those with higher-order derivatives. In the present work, the fourth derivatives of the deflection fields of the innermost and outermost tubes of the DWCNT appear in the equations of motion and it is anticipated that RKPM could reproduce the near to exact values of such deflection fields by using its higher-order interpolant shape functions.

Recently, Kiani [50] examined free transverse vibration of an embedded single-walled tube structure under different boundary conditions. Using various nonlocal beam models and RKPM inerpolants, the flexural frequencies of the nanostructure were obtained and the capabilities of the proposed models in capturing the flexural frequencies of the nanostructure were also studied.

In this article, flexural vibration characterization of a lengthy DWCNT under arbitrary conditions is of concern. The DWCNT is subjected to an initially axial force and is embedded in an elastic matrix. Using Hamilton's principle, the dimensionless equations of motion of the considered DWCNTs are established on the basis of the Rayleigh's beam theory as well as nonlocal continuum theory of Eringen. Since finding an analytical solution to the governing equations is a very difficult task, an efficient meshless technique is proposed. The deflection fields of the innermost and outermost tubes of the lengthy DWCNT are discretized via RKPM, and the governing equations are reconstructed in the matrix form. In some special cases, the obtained results are also compared with those of other works. The roles of the influential parameters on the first five flexural frequencies of lengthy DWCNTs under different boundary conditions are explained and discussed in some detail.

2. Model description

Consider a lengthy elastically supported DWCNT embedded in an elastic medium. A schematic representation of the nanostructure has been shown in Fig. 1. The innermost and outermost tubes are modeled via equivalent continuum structures (ECSs). The ECS is a tubular structure whose length and most of its frequencies are identical to those of the original tube. The mean radius, walls' thickness, length, cross-sectional area, elasticity modulus, and density of the ECS associated