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Love wave in an isotropic homogeneous elastic half-space with a functionally graded cap layer

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ABSTRACT

The dispersion characteristics of Love wave in an isotropic homogeneous half-space covered with a functionally graded layer is investigated. Governing equations for the anti-plane shear wave in the graded layer are derived, and analytical solutions for the displacement and stress field in the layer are given. Moreover, the general dispersion relations of Love wave in both the half-space and the layer are analyzed. For the layer with shear modulus and mass density varying in a parabolic form, the dispersion equations are solved in terms of iteration method. The obtained dispersion curves reveal that there exists a cut-off frequency in the lowest order vibration mode.

1. Introduction

In 1911, Love discovered that when an isotropic homogeneous elastic half-space is covered with a layer having a different elastic medium, there exists a special SH wave in both the covering layer and the half-space. This is known as Love wave. The existence of Love wave successfully explained the dispersion characteristics of the waves that arise in the seismic records. In 1973, Achenbach proved that anti-plane surface waves do not exist on a homogeneous half-space with a free surface [1]. To disclose the dispersion characteristics of the waves in the seismic records, anti-plane shear waves in vertical inhomogeneous medium have also been proposed. Dutta [2,3], Bhattacharya [4], and Chattopadhyay [5] respectively discussed the propagation of Love-type waves in an intermediate non-homogeneous layer lying between two semi-infinite homogeneous elastic media. Chiroiu and Ghiroiu [1] studied propagation of Love waves in an elastic homogeneous half-space covered by an elastic non-homogeneous layer. Based on the Fourier transform method, Abd-Alla and Ahmed [6] studied Love wave dispersion in an initially stressed non-homogeneous orthotropic elastic layer on a semi-infinite medium.

A recent representative paper by Achenbach and Balogun [7] addressed the propagation of anti-plane shear waves in a half-space whose shear modulus and mass density have an arbitrary dependence on the distance from the free surface. By using the WKBJ approximation method, the governing equation was solved; an equation which related the speed of surface waves to the wavenumber was yielded, and the propagation of waves in high-frequency range was detailedly discussed. Liu et al. [8] studied initial stress effect on Love wave in the inhomogeneous piezoelectric covering layer. Li et al. [9] studied the Love wave propagation in functionally graded piezoelectric materials. Collet et al. [10] studied anti-plane surface waves in a functionally graded piezoelectric half-space, and presented an exact solution for the surface wave dispersion spectra without the use of representative layering. An interesting paper on anti-plane shear waves by Shuvalov et al. [11] addressed the determination of anti-plane wave solutions, and discussed general properties of dispersion spectra in a monoclinic plate.

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Based on Biot theory, Ke et al. studied Love waves in inhomogeneous fluid saturated porous layered half-space with varying properties [12,13].

Functionally graded materials, as a kind of typtical inhomogeneous materials, is often used as a layer (or layers) attached on other bulk materials. So anti-plane shear wave in vertical inhomogeneous medium is required to be concerned. Only for several special cases, analytical solution for the governing equation can be found. Even an analytical solution can be obtained, it generally contains extremely complicated functions, making it impractical. So, approximate approaches have to be employed.

In the present work, we study the dispersion properties of Love wave in a homogeneous elastic half-space covered with a functionally graded layer. The dispersion equation is solved by an iterative method, the general properties of Love wave in both media are obtained. The obtained results can be applied to the optimization design of functionally graded materials, nondestructive testing and inverse problem analysis.

2. Governing equations

The geometry of the composite structure is depicted in Fig. 1 along with a Cartesian coordinate system o-*xyz*. Below the half-plane $y \geqslant 0$, an isotropic and homogeneous medium with shear elastic modulus μ and mass density ρ is filled. This medium is covered with a layer of functionally graded materials. The shear elastic modulus and the mass density of the layer are, respectively, μ^B and ρ^B. They are further expressed in the form of $\mu^B = \mu_0^B\mu(y)$ and $\rho^B = \rho_0^B\rho(y)$, where $\mu(y)$, $\rho(y)$ are functions of y, and μ_0^B and ρ_0^B denote the values at $y = 0$. The layer thickness is H. The Love wave propagates along the x direction, and the anti-plane shear motion along the z direction.

2.1. Love wave in homogeneous half-space

We use w to denote the anti-plane displacement in the homogeneous half-space. It takes place along the z direction, satisfies the follwing governing anti-plane shear wave equation

$$\nabla^2 w = \left(\frac{1}{c_T}\right)^2 \frac{\partial^2 w}{\partial t^2}, \quad (0 < y < +\infty), \tag{1}$$

where $c_T = \sqrt{\mu/\rho}$ is the shear wave velocity.

Assuming a traveling wave solution, we have

$$w = Ae^{-by}\exp[ik(x - ct)], \tag{2}$$

where A is a constant that need to be determined, $k = \omega/c$ is the wave number, ω is the angular frequency, c is the phase velocity of Love wave and b is a constant.

Substituting Eq. (2) into Eq. (1), one obtains the value of b,

$$b = k[1 - (c/c_T)^2]^{1/2} \tag{3}$$

and the two shear stress components in the homogeneous half- space:

$$\tau_{xz} = \mu\frac{\partial w}{\partial x} = ik\mu Ae^{-by}\exp[ik(x - ct)], \tag{4a}$$

$$\tau_{yz} = \mu\frac{\partial w}{\partial y} = -b\mu Ae^{-by}\exp[ik(x - ct)]. \tag{4b}$$

2.2. Love wave in functionally graded layer

The anti-plane shear displacement in the functionally graded layer is denoted as w^B, which is also along the z direction, and satisfies

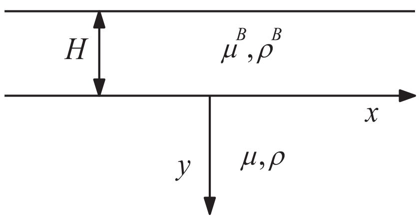

Fig. 1. Geometry of the homogeneous half-space with a graded layer.