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Smoothing Newton method for generalized complementarity problems based on a new smoothing function ☆

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ABSTRACT

In this paper, the generalized complementarity problem is studied. Based on a new smoothing function, the generalized complementarity problem is solved by a smoothing Newton-type algorithm. Under suitable conditions, we prove that the iteration sequence generated by the proposed smoothing method is bounded and the proposed algorithm is globally convergent. Some numerical results are reported.

1. Introduction

The generalized nonlinear complementarity problem (denoted by GCP) is to find a vector $x \in R^n$ such that

$$f(x) \geq 0, \quad g(x) \geqslant 0, \quad f(x)^T g(x) = 0 \tag{1.1}$$

where $f, g : R^n \to R^n$ is continuously differentiable. When $g(x) = x$, GCP reduces to the classic nonlinear complementarity problem (NCP). Moreover, when $f(x) = Mx + q$, with $M \in R^{n\times n}$ and $q \in R^n$, NCP becomes a linear complementarity problem (LCP).

The nonlinear complementarity problem has attracted much attention due to its various important applications. We refer the interested readers to the survey papers [1,2] and references therein.

Many numerical methods for solving NCP have been developed [3–8]. Recently, there has been strong interests in smoothing Newton methods for solving NCP [5,9–13]. The basic idea of smoothing Newton-type methods is to employ a smoothing function to reformulate the problem concerned as a system of smooth equations and then to solve the smooth equations approximately by using Newton's method at each iteration. By making the parameter to tend to zero, one can hope to obtain a solution of the original problem. Lately, Yu and Qin [14] have proposed a cosh-based smoothing Newton method for the nonlinear complementarity problems. The algorithm solves only one linear system of equations, performs only one non-monotone line search per iteration and has global convergence under mild conditions. In this paper, we extend the smoothing Newton-type methods for NCP to solve the generalized nonlinear complementarity problems based on a new smoothing function.

Motivated by the above discussions, we present a new cosh-based smoothing Newton method for GNCP (1.1). The new smoothing NCP-function is proved to possess nice properties. It is testified that our algorithm has the following good properties: (a) We can obtain a solution of (1.1) from any accumulation point of the iteration sequence generated by our algorithm without requiring a priori the existence of an accumulation point. (b) Our algorithm needs only to solve one linear

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system of equations and perform one line search per iteration. (c) Under suitable conditions, our algorithm has global convergence.

The following part of this paper is organized as follows. In the next section, we present some preliminary results and a new cosh-based smoothing Newton method for the GNCP based on the new smoothing function. In Section 3, we establish the global convergence of the proposed algorithm. In Section 4, we report some numerical experiments. Some conclusions are included in Section 5.

The following notions will be used throughout this paper. All vector are column vectors, the subscript T denotes transpose, R^n (respectively, R) denotes the space of n-dimensional real column vectors (respectively, real numbers), R^n_+ (respectively, R^n_{++}) denotes the nonnegative (respectively, positive) orthants of R^n, R_+ (respectively, R_{++}) denotes the nonnegative (respectively, positive) orthants in R. Let $N = \{1, 2, \ldots, n\}$. For any $u \in R^n$, $diag\{u_i, i \in N\}$ denotes the diagonal matrix whose ith diagonal element is u_i and $vec\{u_i, i \in N\}$ the vector u. We use (u, v) for the column vector $(u^T, v^T)^T$. The symbol $\|\cdot\|$ stands for the 2-norm. S denotes the solution set of (1.1).

2. Preliminaries and smoothing method

First, we review some useful definitions and results. Then, we present a new smoothing Newton method for GNCP (1.1).

Definition 2.1. A matrix $M \in R^{n\times n}$ is said to be a P_0-matrix, if all its principal minors are non-negative.

Definition 2.2. A function $F : R^n \to R^n$ is said to be a P_0-function, if for all $x, y \in R^n$ with $x \neq y$, there exists an index $i_0 \in N$ such that $x_{i_0} \neq y_{i_0}$, $(x_{i_0} - y_{i_0})(F(x_{i_0}) - F(y_{i_0})) \geq 0$.

For any $(a, b) \in R^2$, the perturbed minimum function is given by

$$\varphi_\mu(a, b) = \min(a + \mu b, b + \mu a)| = ((1+\mu)(a+b) - |(1-\mu)(a-b)|)/2 \tag{2.1}$$

Obviously, $\varphi_\mu(a, b, \mu)$ is nonsmooth. It is impossible to use Newton-type to solve the given problems. So Huang et al. [11] proposed a smoothing function:

$$\varphi_{CHKS}(\mu, a, b) = (1+\mu)(a+b) - \sqrt{(1-\mu)^2(a-b)^2 + 4\mu^2} \tag{2.2}$$

Based on the idea in [15,14], we can use a Cosh-based smoothing function to approximate the absolute value function, which is given by

$$|x| \approx \varphi(\mu, x) = \mu \ln(2 + 2\cosh(x\mu)) \tag{2.3}$$

where $\cosh(x) = (\exp(x) + \exp(-x))/2$.

Lemma 2.3. *Let $\varphi(\cdot,\cdot)$ be defined by (2.3), then*

(1) $||x| - \varphi(\mu, x)| \leqslant (8\mu/3)\exp(-|x|/\mu)$
(2) *For any $\mu > 0$, $\varphi(\mu, \cdot)$ is twice continuously differential with respect to the second component:*

$$\varphi'(\mu, x) = \sinh(x/\mu)(\cosh(x/\mu) + 1) \in (-1, 1)$$

$$\varphi''(\mu, x) = 1(\mu(\cosh(x/\mu) + 1)) \in (0, 1/2\mu)$$

Proof. The proof can be found in [15], so we can omit it here.

Inspired by the above discussions, we propose a new smoothing function:

$$\varphi(\mu, a, b) = (1+\mu)(a+b) - \mu \ln(2 + 2\cosh((1-\mu)(a-b)/\mu)) \tag{2.4}$$

In the following lemma, we give some properties of the new smoothing function (2.4). □

Lemma 2.4. *Let $\varphi(\cdot,\cdot,\cdot)$ be defined by (2.4), then*

(1) $\lim_{\mu\to 0}\varphi(\mu, a, b) = a + b - |a - b|$.
(2) *for any $(\mu, a, b) \in R_{++} \times R \times R, \partial_a\varphi(\mu, a, b) \in (2\min(\mu, 1), 2\max(\mu, 1))$ and $\partial_b\varphi(\mu, a, b) \in (2\min(\mu, 1), 2\max(\mu, 1))$.*

Proof. By Lemma 2.3, we can easily obtain the result (1). Next, we prove (2). By a simple computation, we have

$$\partial_a\varphi(\mu, a, b) = 1 + \mu - (1-\mu)\sin((1-\mu)(a-b)/\mu)/(1 + \cosh(1-\mu)(a-b)/\mu)$$