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Coalition analysis with preference uncertainty in group decision support

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ABSTRACT

Coalition analysis is extended to incorporate uncertain preference into three stability concepts, general metarationality (GMR), symmetric metarationality (SMR), and sequential stability (SEQ) under the paradigm of the graph model for conflict resolution. As a follow-up analysis in the graph model, coalition analysis aims to assess whether equilibriums under individual calculations are vulnerable to coalition moves and countermoves and, hence, become unstable under coalition stabilities. Coalition analysis has been considered for transitive graph models with simple preference under four stabilities, Nash, GMR, SMR, and SEQ, as well as general graph models with uncertain preference for the Nash stability. This paper introduces preference uncertainty into coalition stabilities under GMR, SMR, and SEQ for general graph models that can be transitive or intransitive. Depending on the focal coalition's different attitudes towards preference uncertainty, four different extensions are presented. Interrelationships of coalition stabilities are investigated within each extension and across the four extensions. A case study is carried out to illustrate how to apply the proposed coalition stabilities.

1. Introduction

Conflict and confrontation among agents with distinct interests may occur at many different settings and scales [3]. To handle strategic conflicts, different approaches have been put forward such as hypergame analysis [7], drama theory [2], and the graph model for conflict resolution [3]. As a simple but flexible group decision technology, the graph model is a proven and invaluable tool for modeling and analyzing strategic conflict in which two or more self-interested agents are in dispute over some issues [3,10]. When a conflict model is established within the graph model framework, two stages are involved: modeling and analysis. In the modeling stage, an analyst or stakeholder identifies two or more decision-makers (DMs) involved in the conflict situation, each DM's available courses of action or options, feasible states formed by all DMs' plausible option selections, state transitions among feasible states controlled by each DM, as well as all DMs' preference over feasible states [3]. Once a conflict model is set up, the analysis stage involves a standard stability analysis and some follow-up analyses such as coalition analysis [11,8,9,22] and status quo analysis [15,16,21]. The stability analysis assesses stability of each state from each DM's perspective and a state that is stable for all DMs is called an equilibrium, corresponding to a potential resolution for the conflict model. The stability analysis is built upon a noncooperative concept with an underlying assumption that each DM acts independently for its own best interests after calculating its moves as well as countermoves by its opponents. Following this line of thinking, the status quo analysis takes a forward looking perspective to assess how DMs act and react to

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direct a conflict from a status quo state or initial state to any particular equilibrium that is of interest to the analyst or stakeholders [16,17,21]. On the other hand, the other post-stability analysis, coalition analysis follows a cooperative viewpoint and assesses whether individual DMs can jointly improve their position by joining a coalition [11,8,22].

Coalition formation and stability have been an active research area in game theory [1,19,13,12]. The coalition analysis considered here is confined to the graph model for conflict resolution paradigm. As Kilgour et al. [11] put it, coalition analysis assesses whether self-interested and independent DMs can gain by forming a coalition and coordinating their moves. This paper follows the idea in [11,22] and treats coalition analysis as a post-stability analysis. The implication is that only equilibria identified in the stability analysis stage will be examined for coalition stability. The rationale is that a non-equilibrium state is not sustainable as at least one DM is expected to deviate from it unilaterally based on the DM's calculations. An equilibrium, on the other hand, is expected to sustain for a while since no DM is motivated to depart from it as per individual contemplations. However, when two or more DMs form a coalition, an equilibrium may be upset via a sequence of joint moves by the coalition members. In this case, the target state should also be an equilibrium as any non-equilibrium state is transient. This process is referred to as an "equilibrium jump" in [11]. Understandably the target state of an equilibrium jump should presumably make all members in the coalition better off and cannot be achieved by any DM acting individually. Coalition analysis, therefore, aims to alert the analyst whether such a coalition exists and, if existent, which equilibria are vulnerable to equilibrium jumps and how these jumps are attained by coalition joint moves.

When a state is assessed for individual stability, different solution concepts such as Nash stability (Nash) [18], general metarationality (GMR) [7], symmetric metarationality (SMR) [7], and sequential stability (SEQ) have been proposed to characterize DMs' distinct behavioural patterns in face of conflict [3]. For details of the characteristics and interrelationships of these solution concepts, readers are referred to Fang et al. [3] and the original references therein.

The original graph model methodology employs a simple preference structure, consisting of strict preference ($\succ$) and indifference ($\sim$) relations, to characterize DMs' relative preference over feasible outcomes. To accommodate the case that some preference information is unknown to the analyst, Li et al. [14] develop a non-probabilistic framework to handle preference uncertainty in the graph model where a new binary relation U is introduced to represent a DM's uncertainty about its preference between two states. The four solution concepts, Nash, GMR, SMR, and SEQ, have been redefined based on the extended preference structure. Depending on how unknown preferences are incorporated, four versions of stability definitions are put forward and labeled as a, b, c, and d accordingly. These different extensions are conceived to reflect the focal DM's distinct attitudes towards preference uncertainty, ranging from conservative, to mixed and aggressive [14].

Within the graph model framework, coalition analysis has been actively studied. Motivated by the strong equilibrium concept by Aumann and Hart [1], Kilgour et al. [11] introduce a coalition Nash stability concept with simple preference and the aim is to alert whether a status quo equilibrium can be upset by joint moves coordinated by a subset of DMs or a coalition. Subsequently, Inohara and Hipel [8] extend the idea and define coalition GMR, SMR, and SEQ stability. The interrelationships of these coalition stabilities are then examined [9]. For tractability, the aforesaid research has been confined to transitive graphs with the simple preference structure, in which consecutive moves by the same DM are allowed. By exploiting a convenient matrix system, Xu et al. [22] investigate coalition Nash stability with preference uncertainty for general graph models, where the requirement of no successive moves by the same DM is honoured to keep the new development consistent with the general decision rule in the graph model methodology. According to how uncertain preferences are incorporated, conservative and aggressive coalition Nash stabilities are introduced [22].

Building upon the research by Xu et al. [22] and Inohara and Hipel [8,9], the contribution of this article is to integrate preference uncertainty into coalition GMR, SMR, and SEQ stabilities. To keep notation consistent with individual stabilities in Li et al. [14], four different versions of each coalition stability will be defined accordingly.

To illustrate how this new development can be applied in practice, a coalition analysis is conducted for a case study of bulk-water export conflict occurred in the Province of Newfoundland and Labrador in Canada. This conflict was first examined by Fang et al. [4] and a three-DM graph model is established to investigate strategic interactions among different stakeholders. Subsequently, preference uncertainty is introduced into the model to characterize the oscillating attitude of the provincial government towards bulk-water export from its jurisdiction [14]. These analyses furnish useful strategic advice on how different stakeholders may act and react to bring the conflict to potential resolutions. The current analysis moves one step further by investigating which equilibria are sustainable and will not be upset by coalition moves and which equilibria are likely to be transient and susceptible to be overturned by a subgroup of DMs coordinating their moves. The aim is to shed additional structural insights on whether any DM may further improve its position by joining a coalition.

To make the paper self-contained, the next section briefly reviews the graph model for conflict resolution and puts the current research in a proper context. Section 3 defines coalition GMR, SMR, and SEQ stabilities with preference uncertainty. Section 4 investigates interrelationships of coalition stabilities within each extension and across the four extensions, followed by an illustrative case study in Section 5. The paper concludes with some remarks in Section 6.

2. Preliminaries

2.1. The graph model for conflict resolution

A graph model consists of a set of DMs $N, 2 \leqslant |N| < \infty$, a finite set of feasible states S, a collection of digraphs $G_i = (S, A_i),\ i \in N$, where S is the vertex set and A_i is DM i's set of directed arcs in G_i and depicts the moves among feasible