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Adaptive pinning cluster synchronization of fractional-order complex dynamical networks

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ABSTRACT

The problem about cluster synchronization of fractional-order CDNs is studied via a pinning adaptive approach in this paper. Based on the stability theory of fractional differential equations, some sufficient criteria for local and global cluster synchronization of fractional-order CDNs are derived. In this paper, the coupling configuration matrix can be asymmetric as well as reducible and the inner coupling matrix can also be asymmetric. Moreover, the number of pinning nodes in each cluster can be evaluated. Especially, when the coupling strength is large enough and the coupling configuration matrix is symmetric, cluster synchronization can be achieved via pinning a single node in each cluster. Finally, some typical examples are given to illustrate the correctness and effectiveness of our results, a surprising finding is that the synchronization performance will become better as the fractional order decreases in this simulation.

1. Introduction

Complex dynamical networks (CDNs) widely exist everywhere and have been studied by researchers in various fields, such as the Internet networks [1], epidemic spreading networks [2] and biological networks [3] etc. As a typical and interesting collective behavior of CDNs, synchronization phenomenon is ubiquitous and has been quite a hot topic due to its potential applications [4,5]. Meanwhile, many different synchronization patterns have been studied in recent years, such as complete synchronization, cluster synchronization, Guaranteed cost synchronization and lag synchronization and so on [6–9].

It is worth noting that Fractional calculus has a history almost as long as conventional calculus. However, its applications in physics and engineering are just a recent subject of interest. Compared with traditional integer-order models, fractional-order ones will be more appropriate to describe memory and hereditary properties of various materials. It has been revealed that many known systems which exhibit fractional dynamics have turned out to be useful in interdisciplinary fields, such as viscoelasticity [10], dielectric polarization [11], electromagnetic waves [12], and quantum evolution of complex system [13]. Not surprisingly, control and synchronization of fractional-order CDNs has currently become one of the most promising research topics since the fractional-order Lorenz system is researched in [14]. Subsequently, several relative results about synchronization of fractional-order CDNs have been proposed [15–17].

In particular, cluster synchronization is more momentous and significant in biological science [18] and communication engineering [19]. As we know, cluster synchronization of CDNs cannot be achieved without control in many real situations, and it will cost too much and always be impractical if all nodes are controlled. To reduce the control cost, only a fraction of

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nodes are controlled, which is known as pinning control. So far, much work about pinning cluster synchronization of integer-order CDNs has been extensively researched in previous literature [20–24].

Nevertheless, to our best knowledge, the problem about pinning cluster synchronization of fractional-order CDNs is seldom studied except [25]. In [25], the problem about pinning cluster synchronization of fractional-order CDNs, in which clusters are divided by additional control rules, is studied. However, different from [25], pinning cluster synchronization problem that clusters are separated in terms of topological structures has never been studied before and the results will be more flexible by adding adaptive law to controllers, which is also one of our motivations. Indeed, it cannot be denied that the desire to study cluster synchronization of fractional-order CDNs has encountered great challenges, since most existing methods about integer-order CDNs cannot be simply extended to fractional-order CDNs. Then the major problem becomes how to find an efficient method to realize cluster synchronization of fractional-order CDNs, which is also one of our motivations.

Motivated by the above discussions, The problem about cluster synchronization of fractional-order CDNs is studied via a pinning adaptive approach in this paper. Based on the stability theory of fractional differential equations, some sufficient criteria for local and global cluster synchronization of fractional-order CDNs are derived. In this paper, the coupling configuration matrix can be asymmetric as well as reducible and the inner coupling matrix can also be asymmetric. Moreover, the number of pinning nodes in each cluster can be evaluated. Especially, when the coupling strength is large enough and the coupling configuration matrix is symmetric, cluster synchronization can be achieved via pinning a single node in each cluster. Finally, some typical examples are given to illustrate the correctness and effectiveness of our results.

2. Model descriptions and preliminaries

2.1. Fraction calculus

To date, there are several definitions about fractional derivatives, such as the Grunwald–Letnikov derivative, the Riemann–Liouville derivative and the Caputo derivative, which are briefly introduced as follows.

The Grunwald–Letnikov derivative with fractional order p is defined by

$$ {}_{a}^{GL}D_t^q f(t) = \lim_{h\to 0} h^q \sum_{r=0}^{[(t-q)/h]} (-1)^{-r} \binom{q}{r} f(t-rh), \tag{1}$$

where $[\cdot]$ means the maximum integer part.

The Riemann–Liouville fractional derivative is defined by

$$ {}_{a}^{R}DD_t^q f(t) = \frac{d^n}{dt^n}\frac{1}{\Gamma(n-q)}\int_a^t \frac{f(\tau)}{(t-\tau)^{q-n+1}}d\tau, \quad n-1<q<n, \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function, $\Gamma(\tau)=\int_0^\infty t^{\tau-1}e^{-t}$.

The Caputo fractional derivative is defined by

$$ {}_{a}^{C}D_t^q f(t) = \frac{1}{\Gamma(n-q)}\int_a^t (t-\tau)^{n-q-1} f^{(n)}(\tau)d\tau, \quad n-1<q<n. \tag{3}$$

It is worth noting that the initial conditions for the Caputo differential derivative take on the same form as for integer-order equations, and in the Caputo derivative there are no restrictions on the values $f^{(s)}(0), s=0,1,2,\ldots,n-1$, which are different from two other fractional differential derivatives. Therefore, the operator ${}_{a}^{C}D_t^q$ is generally called qth-order Caputo differential operator and will be used in the rest of this paper.

2.2. Network model

First, we give a mathematical definition of cluster synchronization.

Definition 1. A network with N nodes is divided into m nonempty subsets $\{G_1,G_2,\ldots,G_m\}$, such as $G_1=\{1,2,\ldots,N_1\}, G_2=\{N_1+1,\ldots,N_2\},\ldots,G_m=\{N_{m-1}+1,\ldots,N_m\}, N_0=0, N_m=N$. A network with N nodes is said to realize cluster synchronization with the partition $\{G_1,G_2,\ldots,G_m\}$, if, for any initial values, the state variables of the nodes satisfy $lim_{t\to+\infty}\|x^i(t)-x^j(t)\|=0$ for $i,j\in G_k$ and $lim_{t\to+\infty}\|x^i(t)-x^j(t)\|\neq 0$ for $i\in G_k$ and $j\notin G_k$.

In this paper, a fractional-order CDN consisting of N nodes is described as the following model:

$$ \frac{d^q x_i(t)}{dt^q} = f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + u_i, \quad i=1,2,\ldots,N, \tag{4}$$

where $0<q<1$ is the fractional order, $x_i(t)=(x_{i1}(t),x_{i2}(2),\ldots,x_{in}(t))\in\mathbb{R}^n$ is the state variable of the ith node, $f:\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^n$ is a smooth nonlinear continuous map, $A=(a_{ij})\in\mathbb{R}^{N\times N}$ denotes the coupling configuration matrix; c is the coupling strength, and $\Gamma\in\mathbb{R}^{n\times n}$ is the inner coupling matrix, $u_i\in\mathbb{R}^n$ are controllers to be designed latter.