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Necessary and sufficient conditions for near-optimal harvesting control problem of stochastic age-dependent system

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ABSTRACT

In this paper, we consider a near-optimal harvesting control problems for stochastic age-structured population system. We establish necessary and sufficient conditions for near-optimality. These conditions are described by adjoint equations and a nearly maximum condition on the Hamiltonian. The proof of the main result is based on Ekeland's variational principle and the adjoint processes with respect to the control variable. As is well known, optimal controls may fail to exist even in simple cases. This justifies the use of near-optimal controls, which exist under minimal assumptions and are sufficient in most practical cases. At last, an example is given for illustrating our results.

1. Introduction and Model

Recently, the near-optimal control problem has become increasingly popular in stochastic differential equations, especially, because it provides an appealing framework for the analysis of control problem, such as Bahlali [1] studied necessary and sufficient conditions for near-optimality in stochastic control of FBSDEs, Huang [2] researched near-optimal control problems for linear forward–backward stochastic system, and near-optimal control for stochastic recursive problems was discussed by Hui [3].

We consider the following stochastic age-dependent system:

$$\begin{cases} \frac{\partial p}{\partial t} + \frac{\partial p}{\partial r} = -u(r,t)p - \mu(r,t)p + f(r,t,p) + g(r,t,p)\frac{dw_t}{dt}, \\ p(0,t) = \beta(t)\int_{r_1}^{r_2} m(r,t)p(r,t)dr, \\ p(r,0) = p_0(r), \\ \bar{p}(t) = \int_0^A p(r,t)dr, \end{cases} \tag{1.1}$$

where $p = p(r,t)$, $Q := (0,A) \times [0,T], t \in (0,T), r \in (0,A); A$ is the life expectancy, and $0 < A < +\infty; [r_1, r_2]$ is the fertility interval.

Though this paper:

$p(r,t)$ denotes the population density of age r at time t;
$\mu(r,t)$ denotes the mortality rate of age r at time t;
$m(r,t)$ is defined as the ratio of females of age r at time t;
$\beta(t)$ denotes the fertility rate of females of age r at time t;
$p_0(r)$ is the initial age distribution;

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$\bar{p}(t)$ is the total population at time t;

$u(r,t)$ is the harvesting effort function, which is the control variable in the model and satisfies: $u(r.t) \in \mathcal{U}_{ad} := \{z \in L^2(Q) : 0 \leqslant \varrho_1 \leqslant z \leqslant \varrho_2 \text{ a.e. in } Q\}$, where $\varrho_1, \varrho_2 \in L^2(Q)$;

$f(r,t,p) + g(r,t,p)\frac{dw_t}{dt}$ denotes the stochastically perturbation, effecting of external environment for population system, such as earthquake, emigration and so on.

For the system (1.1), there have many works studied the existence uniqueness and exponential stability of the solution convergence of numerical solution. Zhang studied the existence, uniqueness and exponential stability numerical solutions problems and convergence of numerical solutions [4–6]. For deterministic system (when $g = 0$), there has been many works on the optimal control problem. For example, Rorres [7] investigated optimal age specific harvesting policy continuous-time population model. For the McKendrick model of population dynamics, Murphy [8] investigated maximum sustainable yield problem for an age-structured population. They obtained optimal solution which was attainable by a bimodal harvesting policy. Chan [9] studied optimal birth control of population systems of McKendrick type which was a distributed parameter system involving first order partial differential equations with nonlocal bilinear boundary control. Anita [10] discussed optimal harvesting for a nonlinear age-dependent population dynamics. Barbu [11] concerned with the optimal control problem for a Gurtin-Mac-Camy type system describing the evolution of an age-structured population. Necessary optimality conditions were established in the form of an Euler–Lagrange system and existence of an optimal control. Anita [12] discussed analysis and control of age-dependent population dynamics. The optimality conditions for age-structured control system were given by Gustav [13].

As we all know, few results were obtained on the topic of near-optimal control problems of stochastic age-structure. The aim of this paper is to study near-optimality. More precisely speaking, the necessary as well as sufficient conditions of near-optimality are established. These conditions are described in terms of an adjoint process, corresponding to the stochastic partial differential equations components and a nearly maximum condition on the Hamiltonian. Our main results are based on the Ekeland's variational principle. This paper is an extension of [4]-[10].

This paper is organized as follows: In Section 2, we give the assumptions, notations, some basic definitions and some Lemmas. In Section 3 and Section 4, we establish necessary and sufficient conditions of near optimality. In Section 5, we provide an example to illustrate our results.

2. Preliminaries of the problem

In this paper, let

$$V = H^1([0,A]) \equiv \left\{\varphi | \varphi \in L^2([0,A]), \frac{\partial \varphi}{\partial x} \in L^2([0,A]),\ \text{where } \frac{\partial \varphi}{\partial x} \text{ are generalized partial derivatives}\right\}.$$

$V' = H^{-1}([0,A])$ is the dual space of V. We denote by $|\cdot|$ and $\|\cdot\|$ the norms in V and V' respectively, by $\langle \cdot,\cdot \rangle$ the duality product between V, V', and by $(\cdot,\cdot)$ the scalar product in H. For an operator $B \in \mathcal{L}(K,H)$ be the space of all bounded linear operators from K into H, we denote by $\|B\|_2$ the Hilbert–Schmidt norm, i.e.

$$\|B\|_2^2 = tr(BWB^T).$$

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a filtrations $\{\mathcal{F}_t\}_{t \geqslant 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets). Let $w(\cdot)$ be a Wiener process defined on the complete probability space and taking its values in the separable Hilbert space K. Let $(\mathcal{F}_t)_{t \geqslant 0}$ be the σ-algebras generated by $\{w_s, 0 \leqslant s \leqslant t\}$, then w_t is a martingale relative to $(\mathcal{F}_t)_{t \geqslant 0}$.

Let $C = C([0,T];V)$ be the space of all continuous function from $[0,T]$ into V with sup-norm $\|\psi\|_C = \sup_{0 \leqslant s \leqslant T} |\psi(s)|$, $L_V^p = L^p([0,T];V)$ and $L_H^p = L^p([0,T];H)$.

Here, without loss of generality, the expected cost on the time interval $[0,T]$ is

$$J(u(r,t)) = E\left[\int_0^T \int_0^A p(r,t)drdt\right] = E\left[\int_0^T \bar{p}(t)dt\right]$$

and the value function is defined as follows:

$$U = \sup_{u(r,t) \in \mathcal{U}_{ad}[0,T]} J(u(r,t)).$$

Since the objective of this paper is to study near-optimal rather than optimal controls of the system, we give the precise definition of near-optimality as given in [2].

Definition 1 (ε-*optimal [2]*). For a given $\varepsilon > 0, u^\varepsilon(\cdot,\cdot)$ is called ε-optimal if

$$|J(u^\varepsilon(\cdot,\cdot)) - U| \leqslant \xi(\varepsilon)$$

holds for sufficiently small ε, where ξ is a function of ε satisfying $\xi(\varepsilon) \to 0$ as $\varepsilon \to 0$, The estimate $\xi(\varepsilon)$ is called an error bound. If $\xi(\varepsilon) = C\varepsilon^\delta$ for some $\delta > 0$ independent of the constant C, then $u^\varepsilon(\cdot,\cdot)$ is called near-optimal with order ε^δ.