Applied Mathematics and Computation 221 (2013) 111–120

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stability analysis of static recurrent neural networks with interval time-varying delay

Jian Sun *, Jie Chen

School of Automation, Beijing Institute of Technology, Beijing 100081, China
Key Laboratory of Complex Systems Intelligent Control and Decision, Beijing Institute of Technology, Beijing 100081, China

A R T I C L E   I N F O

Keywords:
Delay-dependent stability
Interval time-varying delay
Static recurrent neural network
Linear matrix inequality

A B S T R A C T

The problem of stability analysis of static recurrent neural networks with interval time-varying delay is investigated in this paper. A new Lyapunov functional which contains some new double integral and triple integral terms are introduced. Information about the lower bound of the delay is fully used in the Lyapunov functional. Integral and double integral terms in the derivative of the Lyapunov functional are divided into some parts to get less conservative results. Some sufficient stability conditions are obtained in terms of linear matrix inequality (LMI). Numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

During past several decades, recurrent neural networks have been applied in many areas such as speech recognition, handwriting recognition, optimization problem, model identification and automatic control [1,2]. Although neural networks can be implemented by very large scale integrated circuits, there inevitably exist some delays in neural networks due to the limitation of the speed of transmission and switching of signals. It is well known that time-delay is usually a cause of instability and oscillations of recurrent neural networks. Therefore, the problem of stability of recurrent neural networks with time-delay is of importance in both theory and practice. Many results on this topic have been obtained which can be classified into delay-dependent ones and delay-independent ones. Since delay-dependent stability conditions are usually less conservative than delay-independent ones, much attention has been put into developing some less conservative delay-dependent stability conditions [3–24].

Neural networks can be classified into two categories, that is, static neural networks and local field networks. In static neural networks, neuron states are chosen as basic variables. While in local field networks, local field states are chosen as basic variables. It has been proved that these two kinds of neural networks are not always equivalent [25]. Compared with rich results for local field networks, results for static neural networks are much more scare. To mention a few, stability of static recurrent neural networks with constant time-delay was investigated in [26] where new delay-dependent stability criteria were established in the terms of LMI using delay-partitioning approach and Finsler's lemma. By introducing some slack matrices, delay-dependent stability conditions for static recurrent neural networks with time-varying delay were obtained and expressed as LMIs [27]. By constructing a new Lyapunov functional and using s-procedure, both delay-dependent and delay-independent stability conditions were developed for static recurrent neural networks with interval time-varying delays in [28]. Stability and dissipativity analysis of static neural networks were investigated in [29].

* Corresponding author at: School of Automation, Beijing Institute of Technology, Beijing 100081, China.
E-mail addresses: sunjian@bit.edu.cn (J. Sun), chenjie@bit.edu.cn (J. Chen).

http://dx.doi.org/10.1016/j.amc.2013.06.028

In this paper, stability analysis problem of static recurrent neural networks with interval time-varying delays is investigated. As mentioned in our previous works [32,33], information about the lower bound of time-varying delay should be taken into account when constructing a Lyapunov functional. Therefore, a new Lyapunov functional containing some new double-integral terms and triple-integral terms is introduced. Information about the lower bound of delay is more sufficiently used in the Lyapunov functional. Based on the new Lyapunov functional, some less conservative delay-dependent stability conditions are derived. Numerical examples are given to confirm the effectiveness of the proposed method.

Notations. Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes an n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices; $P>0$ means that the matrix P is symmetric positive definite; I is an appropriately dimensional identity matrix.

2. Problem formulation

Consider the following static recurrent neural network with interval time-varying delay:

$$\dot{u}(t)=-Au(t)+g(Wu(t-d(t))+J),\tag{1}$$

where $u(\cdot)=[u_1(\cdot)u_2(\cdot)\cdots u_n(\cdot)]^T$ is the neuron state vector, $A=\mathrm{diag}\{a_1,a_2,\cdots,a_n\}$ with $a_i>0, i=1,2,\cdots,n$, $g(Wu(\cdot))=[g_1(W_1u(\cdot))g_2(W_2u(\cdot))\cdots g_n(W_nu(\cdot))]^T$ is the neuron activation function. $W=[W_1^TW_2^T\cdots W_n^T]^T$ is the delayed connection weight matrix. $J=[j_1,j_2,\cdots,j_n]^T$ is a constant input. $d(t)$ is the time-varying delay and satisfies

$$h_1\leqslant d(t)\leqslant h_2\tag{2}$$

and

$$\dot{d}(t)\leqslant\mu,\tag{3}$$

where $0<h_1<h_2$ and μ are constants.

The following assumption is made in this paper.

Assumption 1. Each bounded neuron activation function, $g_i(\cdot), i=1,2,\cdots,n$ satisfies

$$b_i\leqslant\frac{g_i(s_1)-g_i(s_2)}{s_1-s_2}\leqslant l_i,\quad\forall s_1,s_2\in\mathbb{R}, s_1\neq s_2,\quad i=1,2,\cdots,n,\tag{4}$$

where $b_i,l_i,i=1,2,\cdots,n$ are known real constants.

Assumption 1 guarantees the existence of an equilibrium point of system (1) [30,31]. Denote that $u^*=[u_1^*u_2^*\cdots u_n^*]$ is the equilibrium point. Using the transformation $x(\cdot)=u(\cdot)-u^*$, system (1) can be converted to the following error system:

$$\dot{x}(t)=-Ax(t)+f(Wx(t-d(t))),\tag{5}$$

where $x(\cdot)=[x_1(\cdot)x_2(\cdot)\cdots x_n(\cdot)]^T$ is the state vector, $f(Wx(\cdot))=[f_1(W_1x(\cdot))f_2(W_2x(\cdot))\cdots f_n(W_nx(\cdot))]^T$ with $f(Wx(\cdot))=g(W(x(\cdot)+u^*)+J)-g(Wu^*+J)$. It is easy to see that $f_i(\cdot), i=1,2,\cdots,n$, satisfies

$$b_i\leqslant\frac{f_i(s_1)-f_i(s_2)}{s_1-s_2}\leqslant l_i,\quad f_i(0)=0,\quad\forall s_1,s_2\in\mathbb{R}, s_1\neq s_2,\quad i=1,2,\cdots,n.\tag{6}$$

The following integral inequalities are introduced in the following lemma which is important in the derivation of main results.

Lemma 1 ([32,34]). *For any constant matrix $Z>0$ and scalars $0<h_1<h_2, h_{12}=h_2-h_1$ such that the following integrations are well defined, then*

(1) $-\int_{t-h_2}^{t-h_1}\omega^T(s)Z\omega(s)\mathrm{d}s\leqslant-\frac{1}{h_{12}}\int_{t-h_2}^{t-h_1}\omega^T(s)\mathrm{d}sZ\int_{t-h_2}^{t-h_1}\omega(s)\mathrm{d}s,$

(2) $-\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1}\omega^T(s)Z\omega(s)\mathrm{d}s\mathrm{d}\theta\leqslant-\frac{2}{h_{12}^2}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1}\omega^T(s)\mathrm{d}s\mathrm{d}\theta Z\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1}\omega(s)\mathrm{d}s\mathrm{d}\theta.$

3. Main results

In this section, some new delay-dependent stability criteria are derived by introducing a new Lyapunov functional and using a new method to estimate the derivative of the Lyapunov functional.