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Influence of geometrical distribution of common points on the accuracy of coordinate transformation

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ABSTRACT

Geometrical distribution of common points is of significance for coordinate transformation as the transformation parameters are computed through common points whose coordinates are known. Common points can be divided into a calculating set and a testing set whose geometrical distribution can be described by internal and external distribution parameters. The former contains the number of points, the reference coordinate and the coordinate difference, the latter is defined as the overlapping degree between both sets. Both a rotation matrix and a translation vector are involved in the transformation parameters, and could be evaluated by 3 approaches i.e. the coordinate error method, the RMS error method and the error method of relative Euclidean distance. An invalidation of the third method has been proven that evaluating indexes will remain invariant for a fixed testing set, regardless of variation in the calculating set. According to the classification of geometrical distribution of common points, the influences of internal and external distribution parameters on the accuracy of coordinate transformation are related to the number of points, symmetrical distribution and the overlapping degree, which will be formulated and summarized in detail. Finally, the above conclusions can be verified and proven by computer simulation and practical experiment respectively.

1. Introduction

Coordinate transformation has always been a general problem in several engineering fields, such as geological surveying [1], remote sensing, and large scale metrology, etc. Especially in the manufacturing and assembly industries [2] for automobiles, ships, and aircraft, the complete shape of the mechanical component has to be provided by a few instruments [3] such as theodolite, photogrammetric system and a laser tracker which are suitable to large scale metrology. Owing to the mechanical component with large size and the limit measuring range of instruments, the entire 3D profile of the mechanical component is impossible to obtain through single metrology by only one instrument. To get rid of the dilemma, first, every local shape of the mechanical component can be acquired separately through either placing several instruments on the different stations [4] or changing the different stations by moving the single instrument. Next, all the local features will be integrated into a complete 3D shape through coordinate transformation between two or more coordinate systems. Therefore the involved coordinate transformation is of great importance to ensure higher measuring accuracy of the mechanical component.

The key for such a coordinate transformation is to determine accurate transformation parameters. As the transformation parameters are usually unknown, this matter must be solved by common points whose coordinates are known [5]. According

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to different functions, common points can be classified into two parts: a calculating set and a testing set. The calculating set can be used to compute iterative closed-form transformation parameters repeatedly until a global optimal solution is reached by minimizing the least-squares error criterion. Subsequently, the testing set can be used to evaluate the accuracy of the calculated transformation parameters through 3 approaches i.e. the coordinate error method, the RMS error method and the error method of relative Euclidean distance.

In practice, different solutions of transformation parameters are commonly mixed with error disturbance. Some scholars and experts ascribe the solutions to common points disturbed with errors which are caused by inadequacies of metrology instrument and computation. Hence the mathematical relationship between the errors of transformation parameters and the coordinate errors of common points are studied based on the theory of error propagation in literature [6–8]. Nevertheless, these investigations emphasizing the effect of coordinate errors of common points are not appropriate without considering that of geometrical distribution of common points.

Indeed, geometrical distribution of common points plays an important role in estimating transformation parameters. In a few studies [7,9], the geometrical distribution of the calculating set can be described by the number of points and the distances between points in which the effect of the latter is proved to be more evident than that of the former. In order to improve the accuracy of coordinate transformation, a few suggestions for the reasonable layout of common points are recommended based on many experiences [10], however, they lack systematic formulation and derivation in mathematics.

We are concentrating on the influence of geometrical distribution of common points on the accuracy of coordinate transformation. In this paper, parameter descriptions for common points distribution should be divided into internal and external distribution parameters where the former is composed of the number of points, reference coordinate and the coordinate difference, the latter can be defined as the overlapping degree between both sets. Based on the least squares method, the rotation matrix and the translation vector are computed and evaluated by 3 approaches such as the coordinate error method, the RMS error method as well as the error method of relative Euclidean distance whose invalidation is proven by formula derivation. Related to the number of points, symmetrical distribution and the overlapping degree, the influence of internal and external distribution parameters to coordinate transformation is formulated and summarized in detail. Finally, the above conclusions can be verified through computer simulation and practical experiment independently.

2. Parameter descriptions for geometrical distribution of common points

In the light of the calculating set and the testing set involved in common points, their geometrical distribution can be described by internal distribution parameter and external distribution parameter.

Suppose that there are two coordinate systems C_A and C_B in which the coordinates of any point in the calculating set are $X_{CAi} = (x_{CAi}, y_{CAi}, z_{CAi})^T$ and $X_{CBi} = (x_{CBi}, y_{CBi}, z_{CBi})^T$, $(i = 1,2,\ldots,N)$; similarly, the coordinates of any point in the testing set are $X_{TAj} = (x_{TAj}, y_{TAj}, z_{TAj})^T$ and $X_{TBj} = (x_{TBj}, y_{TBj}, z_{TBj})^T$, $(j = 1,2,\ldots,M)$. Here, the subscripts A and B symbolize two systems, the subscripts C and T denote the point set, N and M limit the number of points of both sets separately. Ideally all the above coordinates have no errors. Two group coordinates of every point in both systems should satisfy the following equations:

$$X_{CBi} = T + R \cdot X_{CAi}, \quad X_{TBj} = T + R \cdot X_{TAj} \tag{1}$$

Equation (1) explains the coordinate transformation from C_A to C_B in accordance with rigid transformation where both parameters are a translation vector T and a rotation matrix R.

2.1. Internal distribution parameter

The internal distribution parameter consists of the number of points, reference coordinates and the coordinate difference. According to the least squares method, transformation parameters can be calculated by no less than 3 points. The more common points there are, the more accurate the parameters will be. With the assumption of common points being rigid body [11], the centroid of the calculating set is considered the reference point for computation of transformation parameters. Reference coordinates, which are defined as the coordinates of the reference point in two coordinate systems, can be shown as $X_{A0} = (x_{A0}, y_{A0}, z_{A0})^T$ and $X_{B0} = (x_{B0}, y_{B0}, z_{B0})^T$. Then the relationship between two group coordinates of the reference point can be written as:

$$X_{B0} = T + R \cdot X_{A0} \tag{2}$$

The coordinate difference indicates the relative position between every other point and the reference point by the coordinate intervals rather than the Euclidean distance. In the coordinate system of C_A, the coordinate difference of any point in the calculating set and that of any point in the testing set should be:

$$X_{cai} = (x_{cai}, y_{cai}, z_{cai})^T = X_{CAi} - X_{A0}, \quad X_{taj} = \left(x_{taj}, y_{taj}, z_{taj}\right)^T = X_{TAj} - X_{A0} \tag{3}$$

Similarly, in the coordinate system of C_B, the coordinate difference of any point in the calculating set and that of any point in the testing set are

$$X_{cbi} = (x_{cbi}, y_{cbi}, z_{cbi})^T = X_{CBi} - X_{B0}, \quad X_{tbj} = \left(x_{tbj}, y_{tbj}, z_{tbj}\right)^T = X_{TBj} - X_{B0} \tag{4}$$