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On non-selective harvesting of a multispecies fishery incorporating partial closure for the populations

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ABSTRACT

A prey–predator type fishery model incorporating partial closure for the populations is described in this paper. The proposed model deals with a problem of non-selective harvesting of a prey–predator system in which both the prey and the predator species obey logistic law of growth. The predator dependent Beddington DeAngelis type functional response is taken into consideration. Dynamic behavior of the system is analyzed. Partial closure for the populations is considered as a controlling instrument to regulate the harvesting of the populations. A dynamic framework towards the optimal utilization of the resource is developed using Pontryagin's maximum principle. The optimal system is numerically solved using an iterative method with Runge–Kutta fourth order scheme. Simulation results show that the optimal control scheme can achieve sustainable ecosystem. Results are analyzed with the help of graphical illustrations.

1. Introduction

The study of resource-management including fisheries, forestry and wildlife management has great importance. Fisheries represent one of the best examples of the exploitation of renewable resources. Fisheries are classified as renewable because the organisms of interest (e.g., fish, shellfish, reptiles, amphibians, and marine mammals) usually produce an annual biological surplus that, with judicious management, can be harvested without reducing future productivity. Fisheries management needs to be formulated addressing the economic and ecological, as well as the temporal and spatial scales of the problem appropriately. Furthermore, a part of the management process, responsibilities of all user-groups and government agencies need to be outlined at all levels. Effective fisheries management requires that managers work towards clearly specified objectives. These may be biological, economic and social. The first stage of the management process is to express the objective as a management strategy. Management action is taken to implement the strategy. Management actions can be divided into catch controls, effort controls and technical measures. Catch controls limit the catches of individual fishers or the fleet as a whole, effort controls limit the numbers of fishers in the fishery and what they can do, while technical measures control the catch that can be made for a given effort.

In this regard, it is necessary to have sustainable use of the resource which means that the resource should be used in such a way that it is neither depleted nor permanently damaged so that the future generation can use that resource. Fishing is sustainable when it can be conducted over the long term at an acceptable level of biological productivity without leading to ecological changes that foreclose options for future generations. Also, there are many cases in which the biological and

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economic productivity of fish populations, ecosystems, and fisheries would be enhanced if the fish populations were allowed to rebuild; this would represent a change from one year to the next. In those cases, rebuilding of those populations and at least some recovery of the ecosystems should be included. Ecosystem based management is an approach that takes major ecosystem components and services – both structural and functional – into account in managing fisheries. It values habitat, embraces a multispecies perspective, and is committed to understanding ecosystem processes. Its goal is to achieve sustainability by appropriate fishery management.

Although modern fishing vessels are often highly sophisticated, it is rare to find a vessel that exploits a single species of fish. These problems of non-selective harvesting have to be taken into account while developing models of multispecies fish harvesting. Problems related to the exploitation of multispecies system are interesting but difficult to be solved both theoretically and practically. Mathematical modelling in harvesting of such fisheries was studied by Clark [1,2], Kot [3], Chaudhuri [4], Kar and Chaudhuri [5,6], Mesterton-Gibbons [7,8], Leard and Rebaza [9] and Chaudhuri and Saha Ray [10]. Such problems of non-selective harvesting cannot however, be removed always by restoring to selective harvesting because selective harvesting itself may generate its own problems. In such cases, large catches from the lower trophic level can have serious implications for production at both levels. Brauer and Soudack [11,12], Dai and Tang [13], Myerscough et al. [14], Xiao and Ruan [15] and some other authors have discussed prey–predator model with harvesting. Various and interesting dynamical behaviors have been observed, such as the stability of equilibria, existence of Hopf-bifurcation, limit cycles, homoclinic loops and even catastrophe. It is also evident that prey–predator interactions in the multispecies system play a crucial role in the management of renewable resources [16–21]. Great attention has been paid to the dynamics properties of the predator–prey models which have significant biological background. Many excellent and interesting results have been obtained in Xu and Ma [22], Gao et al. [23]; Kuang and Takeuchi [24]; Song and Guo [25].

It is reasonable to believe that density dependent factors, like predation, mutual interference etc. plays an important role towards the sustainability of the system. Again, if the abundance of predators is high, density dependent effects start to play an important role towards the stability of the proposed system. In this regard, Beddington DeAngelis type functional response may be considered as the Beddington [26] derived and DeAngelis et al. [27] proposed, independently, a functional response that can accommodate interference among predators (see Huisman and De Boer, [28]). The predator dependent Beddington DeAngelis functional response model can be considered as an extension of the prey-dependent Holling type II functional response model, since it includes, apart from the states searching for prey and handling prey, a third behavioral state, namely mutual interference with competitors. As noted by Skalski and Gilliam [29], predator-dependent functional response can provide better description of predator feeding than prey-dependent functional response over a range of predator–prey abundances and in some cases as in high density population, the Beddington–DeAngelis functional response performed best. Many scholars proposed and studied models consisting of ordinary or functional differential equations incorporating Beddington–DeAngelis type functional response.

To achieve the commercial purpose of the fishery, it is necessary to harvest the population but harvesting should be regulated in such a way that ecological sustainability as well as conservation of the species can be implemented in a long run. In this regard, we have adopted partial closure for the population which means that the entire fishing area is not available for fishing, a fractional part of the entire region is considered as reserve where fishing is strictly prohibited. Partial closure of the population is considered as the control parameter. Subsequently, the optimal control problem is formulated and solved using a numerical iterative method with Runge–Kutta fourth-order scheme. The numerical results provide several realistic features of the model system. Moreover, we have analyzed the results of the optimal controlled system and compared with the results of the same system which is not optimally controlled.

The paper is organized in the following manner. Mathematical model of our considered system is formulated in Section 2. The boundedness of the system is discussed in Section 3. Existence and local stability of the interior equilibrium has been examined in Section 4. Global stability of the system at the interior equilibrium point has been checked in Section 5. Bioeconomic equilibrium of the proposed system is analyzed in Section 6. Optimal harvesting policy is discussed in Section 8 and in Section 9 we have performed numerical simulation of the optimal control problem. A brief conclusion of the biological significance of our findings is also provided in the final section.

2. Model formulation

We consider a prey–predator system with Beddington–DeAngelis type functional response. Let us assume x and y are respectively the size of prey and predator population at time t. It is considered that all these populations are growing in closed homogeneous environment and follow logistic type of growth. Keeping these aspects in view, the dynamical system of the problem may be governed by the following system of differential equations:

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right) - \frac{\alpha xy}{a+bx+cy} - h_1(t), \\ \frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right) + \frac{k\alpha xy}{a+bx+cy} - h_2(t), \end{aligned} \tag{2.1}$$

where $r(K)$ and $s(L)$ are respectively intrinsic growth rate (environmental carrying capacity) of the prey and predator population, α is the maximal relative increase of predation, $k\alpha$ represents the conversion efficiency of consumed prey into new