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Non-parametric probabilistic forecasting of academic performance in Spanish high school using an epidemiological modelling approach

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ABSTRACT

Academic underachievement is a concern of paramount importance in Europe, and particularly in Spain, where around of 30% of the students in the last two courses in high school do not achieve the minimum knowledge academic requirement. In order to analyse this problem, we propose a mathematical model via a system of ordinary differential equations to study the dynamics of the academic performance in Spain. Our approach is based on the idea that both, good and bad study habits, are a mixture of personal decisions and influence of classmates. Moreover, in order to consider the uncertainty in the estimation of model parameters, a bootstrapping approach is employed. This technique permits to forecast model trends in the next few years using confidence intervals. Unfortunately, the obtained results do not suggest improvement in academic performance for the coming years.

1. Introduction

During the last years Spanish authorities have carried out several changes in the educational laws looking for an improvement in academic results [1,2]. These legislative measures have been focused on the educational levels of Compulsory Secondary Education (in the Spanish terminology, Educación Secundaria Obligatoria (ESO)) and the last two courses of high school (in the Spanish terminology *Bachillerato*) that correspond to students between 12–16 and 16–18 years old, respectively. Despite of these efforts, the rates of academic underachievement in these educational stages are still at very worrying levels, about 30% of the pupils [3]. Although less pronounced, these figures are also alarming in many other countries of the European Union [4–7].

The concern about the high level of academic underachievement in Spain is completely justified, not only by the high rates but also by the negative effects on the country's economic development [8], especially in the unemployment and its serious consequences. This issue is of primer importance in the current context of economic crisis affecting particularly Spain.

Nowadays, the job opportunities of people depend on their qualification, their ability to acquire, use and interpret the information, including their skills to adapt the new knowledge to a very demanding and competitive society in constant change. In order to acquire them, students go to schools first and high schools later, learning the contents determined in

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the current legislation. In Spain, the end of the high school courses corresponds to *Bachillerato* and it represents a key-point as we will see later [9].

The access to this academic educational level takes place after finishing the Compulsory Secondary Education (ESO). According to the legislation currently into force [10], the main objective of the academic level of *Bachillerato* is to provide 16–18 years old Spanish students a proper educational training to consolidate the intellectual maturity of the pupils, increasing their specific knowledge as well as boosting the development of abilities that help them to join up either the labour market or higher studies. For all these reasons, this educational level is considered a milestone to students because it represents a period to make important decisions about academic and professional future.

In Spain, students can especialise in three different *Bachillerato* branches: Arts; Sciences and Technology; Humanities and Social Sciences (see Fig. 1). In September, a student of First Stage of *Bachillerato* will transit into the Second Stage of *Bachillerato* if she/he has passed successfully all the subjects except, maybe, at most two out of a total of ten. In order to get the *Bachillerato* degree, it is necessary to pass all the subjects of both *Bachillerato* Stages [2].

According to the different learning theories [11,12], in particular, the Vygotskian perspective [13–15] and the recent studies published [16], habits and behaviour may be socially transmitted, in particular, academic and study habits. Taking into account this approach, in this paper we propose to model the evolution of the academic performance in the educational level of *Bachillerato* in Spain using modelling techniques in mathematical epidemiology. Some examples of analogous situations using type-epidemiological mathematical models are encountered in public health, obesity [17,18], alcoholism [19], drug abuse [20], shopaholism [21], spread of ideas [22], evaluation of law effects on societies [23], and so on.

A first paper following this approach has been published in [24]. In this contribution, bad academic habits are considered as behaviour susceptible to be transmitted among students, mainly between students in the same academic level, on a gender-structured model.

In this paper, we propose a gender-and-course-structured model where we consider, apart from the spread of bad academic habits, the spread of good ones (positive transmission) from students belonging to the promotable group to the non-promotable one. Notice that this additional issue is based on pedagogical strategies that consider mixing groups of students with bad and good academic results in order to induce improvement of them (Educational Inclusion) [25]. Furthermore, we include the estimation of the abandon rates. Abandon is an important aspect still under debate in the pedagogical area which quantification is difficult. We have made a decision in order to include this issue in the model and this is to consider *abandon* when, during the academic year, the student leaves the academic system.

Once the model is stated, we will be able to monitor the promoted and graduated students. Other new contribution is the introduction of uncertainty in the obtention of the value of the parameters of our model which will allow us to predict the evolution of the academic performance in specific confidence intervals.

The proposed approach will allow us to understand better the mechanism behind the academic performance as well as to predict how things will evolve in the Spanish *Bachillerato* over the next few years and this way, to provide relevant information to make appropriate decisions to policymakers.

This paper is organised as follows. In Section 2, we build and introduce the model. Once the model is stated, unknown parameters are estimated. We present the parameter estimation procedure in Section 3. In Section 4, uncertainty in model parameters is introduced using bootstrapping technique and predictions over the next few years about academic performance in *Bachillerato* are presented. Section 5 deals with the quantification and analysis of abandon. Conclusions are given in Section 6.

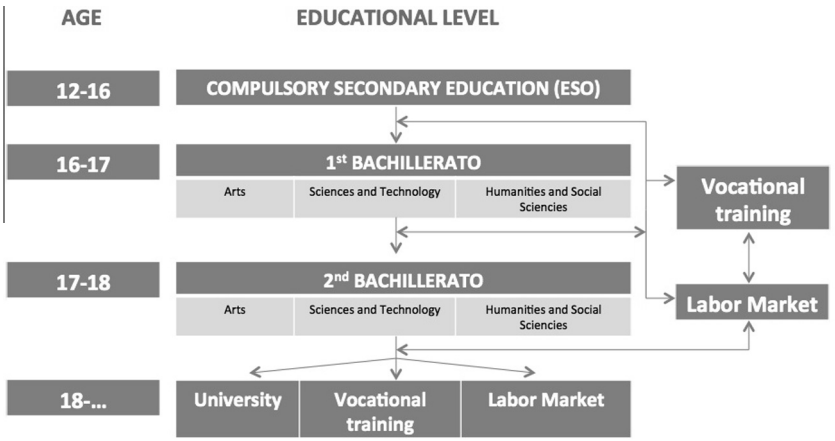

Fig. 1. Structure of the Spanish educational system for students aged 12–18. Most of the students follow the academic path ESO + *Bachillerato*.