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Exponential passivity of BAM neural networks with time-varying delays

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ABSTRACT

The problem of delay-dependent exponential passivity analysis is investigated for BAM neural networks with time-varying delays. By establishing a suitable augmented Lyapunov–Krasovskii function and a novel sufficient condition is obtained to guarantee the exponential stability of the considered system. The several exponential stability criterion proposed in this paper is simpler and effective. A numerical example is provided to demonstrate the feasibility and effectiveness of our results.

1. Introduction

It is well known that bidirectional associative memory (BAM) neural networks were originally introduced by Kosko [1–3]. This class of neural network has been widely studied both in theory and applications. Especially, in practical applications, the BAM neural network has been successfully applied to image processing, pattern recognition, automatic control, associative memory, parallel computation and optimization problems. Therefore, it is meaningful and important to study the BAM neural network. Due to their extensive practical applications, considerable attention has been devoted to BAM neural networks. The issues of robust passivity, delay-dependent and stability have been well investigated; see, for example, [12,13,20–27,31] and references therein. Moreover, the issues of synchronization and dissipativity of neural networks were proposed in [4,28–30].

On the other hand, the passivity theory was firs related to the circuit theory plays an important role in the analysis and design of linear and nonlinear systems, especially for high-order systems [9]. It should be pointed out that the essence of the passivity theory is that the passive properties of a system can keep the system internal stability. Very recently, the exponential passivity of neural networks with time-varying delays has been studied in [10], where sufficient conditions have been obtained for the considered neural networks to be exponential passivity. It is worth pointing out that exponential passivity implies passivity, but the converse does not necessarily hold. However, the information of neuron activation functions and the involved time-varying delays has not been adequately considered in [10,11], which may lead to conservatism to some extent. In [6,14–19], the passivity analysis of neural networks was studied, and, the issues of uncertainty and discrete-time have been proposed. In [5], Wu et al. studied the new results on exponential passivity of neural networks with time-varying delays. However, to the best of our knowledge, the problems of exponential passivity of BAM neural networks with time-varying delays has not yet been investigated, which motivates this work.

In this paper, the problem of delay-dependent exponential passivity analysis for BAM neural networks with time-varying delay is considered. By constructing a new Lyapunov–Krasovskii functional which fractions delay interval and employing different free-weighting matrices in the upper bounds of integral terms, a novel class of Lyapunov functional is constructed to derive some novel delay-dependent stability criteria. It is shown that this obtained conditions have less conservatism. Finally, a numerical example is given to show the usefulness of the proposed criteria.

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2. Model description and Preliminaries

Consider the BAM neural networks with time-varying delays described by the following state equations:

$$\begin{cases} \dot{x}(t) = -Ax(t) + Cf(y(t)) + Ef(y(t-h(t))) + u(t) \\ z_1(t) = f(y(t)) + f(y(t-h(t))) + u(t) \\ \dot{y}(t) = -By(t) + Dg(x(t)) + Fg(x(t-\tau(t))) + v(t) \\ z_2(t) = g(x(t)) + g(x(t-\tau(t))) + v(t) \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathsf T}$ and $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^{\mathsf T}$ are state vectors associated with the n neurons; $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^{\mathsf T}$ and $f(y(t)) = [f_1(y_1(t)), f_2(y_2(t)), \ldots, f_n(y_n(t))]^{\mathsf T}$ are the neuron activation functions; $A = diag(a_1, a_2, \ldots, a_n)$ and $B = diag(b_1, b_2, \ldots, b_n)$ are diagonal matrices with positive entries $a_i > 0$ and $b_i > 0$; C and D are the connection weight matrices, E and F are the delayed connection weight matrices. $u(t)$ and $v(t)$ are the external input vector to neurons, $z_1(t)$ and $z_2(t)$ are the output vector of neuron networks. The delays $h(t)$ and $\tau(t)$ are time-varying satisfying

$$0 \leqslant \tau(t) \leqslant \tau_u, \quad 0 \leqslant h(t) \leqslant h_u, \quad \dot{\tau}(t) \leqslant \tau_D < 1, \quad \dot{h}(t) \leqslant h_D < 1.$$

Throughout this paper, we make the following assumption, definition and lemmas:

(Assumption) There exists two positive diagonal matrices $L = diag(l_1, l_2, \ldots, l_n)$ and $H = diag(h_1, h_2, \ldots, h_n)$ such that $0 \leqslant \frac{f_i(x)-f_i(y)}{x-y} \leqslant l_i$ and $0 \leqslant \frac{g_i(x)-g_i(y)}{x-y} \leqslant h_i$ for any $x, y \in \Re, x \neq y$, and $f_i(0) = 0, g_i(0) = 0, i = 1, 2, \ldots, n$.

Definition 2.1. The BAM neural networks are said to be exponentially passive from input, if there exists an exponential Lyapunov function $V(x_t, y_t)$, and a constant $\rho > 0$ such that for all $u(t)$ and $v(t)$, all initial conditions $x(t_0)$ and $y(t_0)$, all $t \geqslant t_0$, the following inequality holds:

$$\dot{V}(x_t, y_t) + \rho V(x_t, y_t) \leqslant 2(z_1^{\mathsf T}(t)u(t) + z_2^{\mathsf T}(t)v(t)), \quad t \geqslant t_0, \tag{2}$$

where $\dot{V}(x(t), y(t))$ denotes the total derivative of $V(x(t), y(t))$ along the state trajectories $x(t)$ and $y(t)$ of system (1).

The parameter ρ provides an exponential convergence information about an upper bound of exponential Lyapunov function. If ρ increases, then tighter bound about Lyapunov function than the results in case of $\rho \geqslant 0$ can be provided.

Lemma 2.1. (*Schur complement [7]*). *Given constant matrices Z_1, Z_2, Z_3, where $Z_1 = Z_1^{\mathsf T}$ and $Z_2 = Z_2^{\mathsf T} > 0$. Then $Z_1 + Z_3^{\mathsf T} Z_2^{-1} Z_3 < 0$ if and only if $\begin{bmatrix} Z_1 & Z_3^{\mathsf T} \\ Z_3 & -Z_2 \end{bmatrix} < 0$ or $\begin{bmatrix} -Z_2 & Z_3 \\ Z_3^{\mathsf T} & Z_1 \end{bmatrix} < 0$.*

Lemma 2.2. *For all real vectors a, b and all matrix $Q > 0$ with appropriate dimensions, it follows that:*

$$2a^{\mathsf T}b \leqslant a^{\mathsf T}Qa + b^{\mathsf T}Q^{-1}b$$

Lemma 2.3. *For any scalar $h(t) \geqslant 0$, and any constant matrix $Q \in \Re^{n\times n}, Q = Q^{\mathsf T} > 0$, the following inequality holds:*

$$-\int_{t-h(t)}^{t} \dot{y}^{\mathsf T}(s)Q\dot{y}(s) \leqslant h(t)\xi_1^{\mathsf T}(t)\chi Q^{-1}\chi^{\mathsf T}\xi_1(t) + 2\xi_1^{\mathsf T}(t)\chi[y(t) - y(t-h(t))] \tag{3}$$

where $\xi_1(t) = [y^{\mathsf T}(t) \;\; y^{\mathsf T}(t - \frac{h_u}{3}) \;\; y^{\mathsf T}(t - \frac{2h_u}{3}) \;\; y^{\mathsf T}(t-h_u) \;\; y^{\mathsf T}(t-h(t)) \;\; \dot{y}^{\mathsf T}(t) \;\; f^{\mathsf T}(y(t)) \;\; f^{\mathsf T}(y(t-h(t))) \;\; v^{\mathsf T}(t)]^{\mathsf T}$ and χ is free-weighting matrix with appropriate dimensions.

3. Main results

In this section, we propose a new exponential passivity criterion for the BAM neural networks with time-varying delays system. Now, we have the following main results.

Theorem 3.1. *For given scalars $h_u > 0, \tau_u > 0, \rho > 0, \tau_D < 1$ and $h_D < 1$, the delayed BAM neural networks (1) is exponential passive, if there exist positive diagonal matrices $R_i = diag(r_{i1}, r_{i2}, \ldots, r_{in})(i = 1, 2)$, positive definite matrices P_j, K_j $(j = 1, 2), M_i(i = 1, 2, 3, 4, 5, 6)$, $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}$, $\begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}$ and any matrices $N_i, T_i \;\; (i = 1, 2, 3, 4), X_j, Y_j, Z_j, W_j, \tilde{X}_j, \tilde{Y}_j, \tilde{Z}_j, \tilde{W}_j (j = 1, 2, 3, 4, 5, 6)$ with appropriate dimensions such that the following LMIs hold:*