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Persistence and extinction in general non-autonomous logistic model with delays and stochastic perturbation ☆

Chun Lu [a,b], Xiaohua Ding [a,*]

[a] Department of Mathematics, Harbin Institute of Technology, Weihai 264209, China
[b] Department of Mathematics, Qingdao Technological University, Qingdao 266520, China

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ABSTRACT

This is a continuation of the paper Liu and Wang (2012) [14]. Firstly, we consider a general non-autonomous logistic model with delays and stochastic perturbation. Then sufficient conditions for extinction are established as well as nonpersistence in the mean, weak persistence and stochastic permanence. The threshold between weak persistence and extinction is obtained. Finally, numerical simulations are introduced to support the theoretical analysis results.

1. Introduction

A classic non-autonomous logistic model with time-varying and infinite delays can be expressed as follows

$$dx(t)/dt = x(t)\left[r(t) - a(t)x(t) + b(t)x(t-\tau(t)) + c(t)\int_{-\infty}^{0} x(t+\theta)d\mu(\theta)\right]dt, \tag{1.1}$$

where $\tau(t) \geqslant 0$ represents the time-varying delay and $\mu(\theta)$ is a probability measure on $(-\infty, 0]$. A further and extensive feature is considered in model (1.1) or systems similar to (1.1) towards persistence, extinction, attractivity or other dynamical system. Here, we only refer to Gopalsamy [1,2], Kuang and Smith [3], Wang and Wang [4], Li et al. [5], Gakkhar and Singh [5], He and Gopalsamy [6], Lisena [7] and Kuang [8]. Particularly, Gopalsamy [1] and Kuang [8] are all good references in this field.

In the real world, population models are always influenced by environmental noises (see e.g., [9,10]). Moreover, May [11] has revealed the fact due to environmental noise, the birth rate, carrying capacity, competition coefficient and other parameters involved in the system of exhibit random fluctuation to a greater or lesser extent. Inspired by works referred above, we estimate the birth rate $r(t)$ and the intraspecific competition coefficient $-a(t)$ by an average value plus errors which follow a normal distribution. In other words, we may substitute the parameters $r(t),\ -a(t)$ with $r(t) + \sigma_1(t)\dot{\omega}_1(t),\ -a(t) + \sigma_2(t)\dot{\omega}_2(t)$, respectively. Here $\sigma_i(t)(i=1,2)$ are continuous positive bounded function on $\overline{R}_+ = [0, +\infty)$ and $\sigma_i^2(t)(i=1,2)$ represents the intensity of the white noise at time t; $\dot{\omega}_i(t)(i=1,2)$ are the white noises, namely $w_i(t)(i=1,2)$ is a Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t\in\overline{R}_+}$ satisfying the usual conditions (i.e., it is right continuous and increasing while $\mathcal{F}_0$ contains all $\mathcal{P}$-null sets). Then we obtain the following model:

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* Corresponding author.
E-mail addresses: mathlc@126.com, mathlc@163.com (X. Ding).

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$$dx(t) = x(t)\left[r(t) - a(t)x(t) + b(t)x(t-\tau(t)) + c(t)\int_{-\infty}^{0} x(t+\theta)d\mu(\theta)\right]dt + \sigma_1(t)x(t)dw_1(t) + \sigma_2(t)x^2(t)dw_2(t). \quad (1.2)$$

In this paper, we shall investigate the following general non-autonomous logistic model with time-varying delay, infinite delay and stochastic perturbation:

$$dx(t) = x(t)\left[r(t) - a(t)x(t) + b(t)x(t-\tau(t)) + c(t)\int_{-\infty}^{0} x(t+\theta)d\mu(\theta)\right]dt + \sigma_1(t)x(t)dw_1(t) + \sigma_2(t)x^{2+\kappa}(t)dw_2(t), \quad (1.3)$$

where κ satisfies the following conditions (**A2**). Let the initial data ξ be positive and belong to the friendly spaces $C_{\mathbf{r}}$ which defined by

$$C_{\mathbf{r}} = \{\varphi \in C((-\infty,0];(0,+\infty)) : \|\varphi\|_{c_{\mathbf{r}}} = \sup_{-\infty<\theta\leqslant 0} e^{\mathbf{r}\theta}|\varphi(\theta)| < +\infty\},$$

where $\mathbf{r} > 0$. It is easy to verify that $C_{\mathbf{r}}$ is an admissible Banach space (see [12,13]). And μ is the probability measure on $(-\infty, 0]$ satisfying that

$$\mu_{\mathbf{r}} = \int_{-\infty}^{0} e^{-2\mathbf{r}\theta}d\mu(\theta) < +\infty. \quad (A1)$$

Obviously, the above assumption is satisfied when $\mu(\theta) = e^{k\mathbf{r}\theta}(k > 2)$ for $\theta \leqslant 0$, hence there exists a large number of these probability measures.

Things must be pointed out that a considerable number of classical and important stochastic models are the particular cases of model (1.3). For example, if $b(t) = 0,\ c(t) = 0,\ \sigma_2(t) = 0$, then model (1.3) becomes classic stochastic non-autonomous logistic model; if $\kappa = 0$, then model (1.3) takes the form of the model (1.2). Above all, Liu and Wang (see Ref. [14]) pointed out that an interesting problem is to consider the permanence and extinction of the following model:

$$dx_i(t) = x_i(t)\left[r_i + \sum_{j=1}^{n} a_{ij}x_j(t) + \sum_{j=1}^{n} b_{ij}x_j(t-\tau_{ij}) + \sum_{j=1}^{n} c_{ij}\int_{-\infty}^{0} x_j(t+\theta)d\mu_{ij}(\theta)\right]dt + \sigma_i x_i(t)(x_i(t) - x_i^*)dB_i(t), \quad 1 \leqslant i \leqslant n, \quad (1.4)$$

where $(x_1^*, \ldots, x_n^*)$ is an positive equilibrium state of model (1.4).

Motivated by the work of Liu and Wang [14], we will investigate the persistence and extinction of model (1.3). As far as we are concerned, there are few results of this aspect for model (1.3). Furthermore, up to the authors' knowledge, all the publications have not obtained the persistence-extinction threshold for model (1.3). The principle aim of this paper is to explore the problems above. Simultaneously, it can also generalize the work of Marion et al. [15] on the uniqueness of the positive and global solution, and extend the commitments of Liu and Wang (see, [14,16]).

For system (1.3) we always assume:

(**A2**): $r(t), a(t), b(t)$ and $c(t)$ are continuous and boundeded function on $\overline{R}_+$ and $\inf_{t\in\overline{R}_+} a(t) > 0$. Moreover, $\kappa \geqslant -0.25$.
(**A3**): $\tau(t)$ is continuously differentiable function with $0 \leqslant \tau(t) \leqslant \tau^M$ and $1 - \dot{\tau}(t) > 0$ for $t \in R$, where τ^M is a constant. $\sigma^{-1}(t)$ is inverse function of $\sigma(t) = t - \tau(t)$.

For the simplicity, we define the following notations:

$$f^u = \sup_{t\in R} f(t), \quad f^l = \inf_{t\in R} f(t), \quad \langle x(t)\rangle = \frac{1}{t}\int_0^t x(s)ds,$$

$$x_* = \liminf_{t\to+\infty} x(t), \quad x^* = \limsup_{t\to+\infty} x(t), \quad R_+ = (0,+\infty),$$

$$g^* = \limsup_{t\to+\infty} t^{-1}\int_0^t \left(r(s) - \frac{\sigma_1^2(s)}{2}\right)ds.$$

The following definitions are commonly used and we list them here.

Definition 1.1

1. The population $x(t)$ is said to go to extinction if $\lim_{t\to+\infty} x(t) = 0$.
2. The population $x(t)$ is said to be nonpersistence in the mean (see e.g., [17]) if $\limsup_{t\to+\infty}\langle x(t)\rangle = 0$.
3. The population $x(t)$ is said to be weak persistence (see e.g., [18]) if $\limsup_{t\to+\infty} x(t) > 0$.
4. Population size $x(t)$ is said to be stochastic permanence if for arbitrary $\varepsilon > 0$, there are constants $\beta > 0, M > 0$ such that $\liminf_{t\to+\infty}\mathcal{P}\{x(t) \geqslant \beta\} \geqslant 1 - \varepsilon$ and $\liminf_{t\to+\infty}\mathcal{P}\{x(t) \leqslant M\} \geqslant 1 - \varepsilon$.