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Bifurcation analysis of the generalized stretch-twist-fold flow

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ABSTRACT

Based on the stretch-twist-fold flow, a generalized stretch-twist-fold flow is introduced. By choosing an appropriate bifurcation parameter, Hopf bifurcations occur in this system when the bifurcation parameter exceeds a critical value. The formulae for determining the direction of the Hopf bifurcations and the stability of bifurcating periodic solutions are presented. In addition, the paper also investigates the bifurcations of the heteroclinic orbits for this system. The existence and its associated existing regions are given for two heteroclinic orbits, respectively. Finally, some numerical simulations for justifying the theoretical analysis are presented.

1. Introduction

Since the discovery of the Lorenz chaotic system, chaos has been developed and intensively studied in the past four decades. Recently this study about chaos has concentrated on not only proposing new and interesting chaotic systems, but also enhancing complex dynamics and topological structure based on the existing chaotic systems [1–6].

The stretch-twist-fold (STF) was devised to represent the stretch-twist-fold mechanism of the magnetic field generation that is believed to be most conducive to the fast dynamo action in magnetohydrodynamics [7,8]. Although there is quite extensive literature about the STF flow [9–13], up to now nothing has been known about the bifurcations of the STF flow. Also, Hopf bifurcations do not exist in the system. To further investigate the bifurcations of the system, we introduce a new system containing the STF flow, which not only preserves the original dynamic properties, but also possesses some new characteristics so as to benefit systematic studies and help reveal the most essential dynamical behavior of the classic STF flow. In addition, since the stretch-twist-fold action represents the optimal reinforcement of stretched field available in a three-dimensional domain [7], the results from the new system are possibly helpful for finally exposure of a magnetic field structure in a plasma contained in a domain bounded by a perfect conductor.

In this paper, by introducing two parameters to the original STF flow, we propose a generalized stretch-twist-fold flow. It is interesting to find that the control parameters to the original STF flow, may be able to change its some dynamic properties. Hopf bifurcations do not exist in the original STF flow, but they exist in the generalized STF flow. The original STF flow is conservative whereas the generalized STF flow is no longer conservative. The original STF flow is a class of three-dimensional incompressible steady flows in a sphere, but the generalized STF flow may be not in a sphere when the parameters vary. We also investigate the heteroclinic bifurcations of the new system, which include the heteroclinic bifurcations of the original system as a special case.

By choosing an appropriate bifurcation parameter, the paper proves that Hopf bifurcations occur in the generalized STF flow when the bifurcation parameter exceeds a critical value and presents the formulae for determining the direction of the Hopf bifurcation and the stability of bifurcating periodic solutions by applying the normal form theory [14,15]. We also study

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the bifurcations of the heteroclinic orbits by the generalized Melnikov method [16]. The existence and its associated existing regions are given for two heteroclinic orbits, respectively. Finally, some numerical simulations are performed to justify the theoretical analysis.

The rest of this paper is organized as follows. In Section 2, we present the generalized stretch-twist-fold flow and report the existence of chaos with different initial conditions. In Section 3, by using the normal form theory, the direction of Hopf bifurcations and the stability of bifurcating periodic solutions are analyzed in detail. Section 4 explores the bifurcations of the heteroclinic orbits by the generalized Melnikov method. In Section 5, some numerical simulations are presented to illustrate the theoretical analysis. And Section 6 concludes the paper.

2. The generalized stretch-twist-fold flow

By introducing two parameters to the original STF flow [9], the following generalized stretch-twist-fold flow is obtained:

$$\begin{cases} \frac{dx}{dt} = \alpha z - 8(b+1)xy, \\ \frac{dy}{dt} = 11x^2 + 3y^2 + z^2 + \beta xz - 3c^2, \\ \frac{dz}{dt} = -\alpha x + 2yz - \beta xy, \end{cases} \tag{2.1}$$

where b, c, α and β are real parameters, determining the chaotic behaviors and bifurcations of the system. In the original STF flow, α and β are positive real parameters and related to the ratios of intensities of the stretch, twist and fold ingredients of the flow.

For $\alpha \geq 0$ and $\beta \geqslant 0$, system (2.1) always has two isolated equilibria $P_+[0, c, 0]$ and $P_-[0, -c, 0]$. When $\alpha > 0$, there are other equilibria. This system is invariant under the transformation

$$(x, y, z) \to (-x, y, -z),$$

namely, the system has rotation symmetry around the y-axis. The divergence of system (2.1) is $\nabla \cdot V = -8by$, so the system is no longer conservative.

When the parameters $\beta = 1.5,\ b = 0.001,\ c = 1.7$ and $\alpha = 0.1$, the three Lyapunov exponents with initial values (0.2, 0.5, 0.5) of system (2.1) are $L_1 = 0.2246,\ L_2 = -0.0025,\ L_3 = -0.2220$, and the corresponding Lyapunov exponents spectrum for $\alpha \in [5.3, 7]$ are shown in Fig. 1.

When the parameters $\beta = 1,\ b = 0.1,\ c = 1$ and $\alpha = 10000$, the Poincaré map with initial values (0.33, 0.5, 0.3) also shows that this system is chaotic, as shown in Fig. 2.

3. Hopf bifurcations in system (2.1)

In this section, we employ the normal form theory [14,15] to study the direction, stability and period of bifurcating periodic solutions for system (2.1).

System (2.1) always has two isolated equilibria: $P_-[0, -c, 0]$ and $P_+[0, c, 0]$ for any parameters α, β and b. The eigenvalues of the equilibrium $P_-[0, -c, 0]$ are

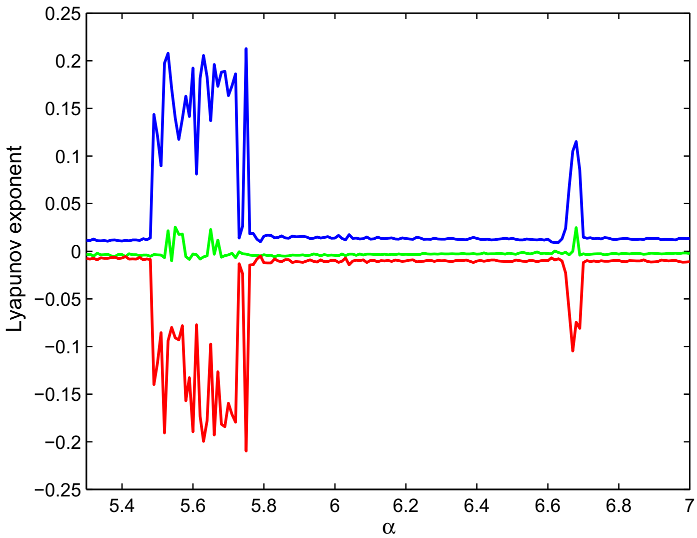

Fig. 1. Lyapunov exponent spectrum of system (2.1) for $\alpha \in [5.3, 7]$ with parameters values (β, b, c) = (1.5, 0.001, 1.7).