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Chaos synchronization between the coupled systems on network with unknown parameters

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ABSTRACT

This work investigates the adaptive network synchronization of different coupled chaotic (or hyper-chaotic) systems with unknown parameters. The definition of network synchronization is presented between the coupled systems where there are 2 N chaotic systems, the former N systems are coupled each other and so are the later N systems. Then the sufficient conditions for achieving this synchronization are derived based on the theory of the stability theory of dynamical systems with 2 N coupled chaotic systems. By the adaptive control technique, the control laws and the corresponding parameters update laws are proposed such that this network synchronization of non-identical chaotic (or hyper-chaotic) systems is to be obtained. These results obtained are in good agreement with the existing one in open literature and it is shown that the technique introduced here can be further applied to various network synchronizations between coupled dynamical systems. Finally, all our analytic results are confirmed by extensive numerical simulations of the model.

1. Introduction

It has been repeatedly demonstrated by scientists in the last recent decades that nonlinear systems, which models our real world, can display a variety of behaviors including chaos and hyperchaos. We could try controlling chaos for the benefit of our applications. Synchronization or anti-synchronization of different chaotic or hyperchaotic systems has been one of the few main control methods which are popularly discussed for several years. In recent years, there has been an dramatic attention among the scientists of mathematics, physics and engineering fields in the study of chaos synchronization between chaotic systems, due to its useful applications in secure communication, power convertors, biological systems, information processing and chemical reactions [1]. The main idea of synchronization between chaotic systems is to design a suitable controller to control the response system such that the response system states track the master system states asymptotically. Until now, a lot of different control schemes have been developed for synchronization of chaotic systems, such as linear feedback method [2], adaptive control[3,4], impulsive method [5], stochastic control [6], H_∞ approach [7], fuzzy logic control [8], etc. Most of the works about synchronization have assumed that the parameters of chaotic systems are known beforehand. However, the parameters of chaotic systems are inevitably perturbed by external factors in many real world applications and the values of them cannot be exactly known in advance. Therefore, investigation of synchronizing two chaotic systems with fully unknown parameters has become a significant topic. Thus, some control schemes have presented for chaos synchronization [9–11]. In particular, a necessary and sufficient condition for proper parameter identification which is crucial for many practical applications in [12].

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It is increasingly recognized that organizing principles operate in most real networks [12]. Many social, biological, and information systems can be well described by networks, where nodes represent individuals, biological elements (proteins, genes, etc.), computers, web users, and so on, and links denote the relations or interactions between nodes. The study of complex networks has therefore become a common focus of many branches of science [13]. Great efforts have been made to understand the evolution of networks [14], the relations between topologies and functions [15,16], and the network characteristics [17]. The synchronization of all dynamical nodes is one of the most significant and interesting properties in a complex network. Therefore, the synchronization problem for complex dynamical networks has received increasing research attention, and a great deal of results have been available in the literature [18–21].

In this letter, we propose a generalized adaptive network synchronization scheme based on the stability theory with fully unknown parameters. There are 2 N chaotic (or hyperchaotic) systems on network. The former N node systems are coupled each other and so are the later N node systems. The error systems of the network synchronization are constructed between the former N nodes and the later N nodes. Then we use adaptive control to implement this particular kind of synchronization with unknown parameters. Based on the above reason and the so-called lyapunov stability conditions for the continuous systems, we describe this kind of synchronization as generalized network synchronization with unknown parameters, which is less restrictive but more practical.

The rest of this paper is organized as follows. The definition and the main results for the network synchronization are provided in Section 2. An illustrative example is presented to show the effectiveness of the obtained scheme in Section 3. Conclusion and discussion are finally given in Section 4.

2. Generalized network synchronization between the coupled chaotic (or hyperchaotic) systems

Consider the 2 N chaotic (or hyperchaotic) systems, the first N systems are described by

$$\dot{x}_i = f_i(x_i) + F_i(x_i)P_i + \sum_{k=1}^{N} A_{ik}x_k \quad i = 1,2,...,N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, ..., x_{im_i})^T \in R^{m_i}$ is the ith state vector of the drive systems, m_i is positive integer, $f_i(x) \in C(R^{m_i}, R^m)$ including nonlinear terms, $F_i \in C(R^{m_i}, R^{m_i \times l_i})$, $P_i \in R^{l_i}$ as the vector of system parameters and A_{ik} is $m_i \times m_k$ matrix. The other N response systems with controllers are given by

$$\dot{x}_{N+i} = f_{N+i}(\dot{x}_{N+i}) + F_{N+i}(x_{N+i})P_{N+i} + \sum_{k=1}^{N} A_{N+i,N+k}x_{N+i} + u_{N+i}, \quad i = 1,2,..,N, \tag{2}$$

where $x_{N+i} = (x_{N+i,1}, x_{N+i,2}, ..., x_{N+i,n_{N+i}})^T \in R^{n_i}$ is the i-th state vector of the response systems, n_i is positive integer, $f_{N+i} \in C(R^{n_i}, R^{n_i})$ including nonlinear terms, $F_{N+i} \in C(R^{n_i}, R^{n_i \times l_{N+i}})$, $P_{N+i} \in R^{l_{N+i}}$ as the parameter vector of the response system, $A_{N+i,N+k}$ is $n_i \times n_k$ matrix and $u_{N+i} \in R^{n_i}$ is the controller. $\Omega_i \in R^{h_i \times h_i}$ is the positive definite matrix with the suitable order, $Q_i \in R^{h_i \times m_i}$ and $S_i \in R^{h_i \times n_i}$ (i = 1,2,...,N). Then the purpose is to design a controller $u(u \in R^{n_i})$ which is able to synchronize the 2 N chaotic (hyper-chaotic) systems with identical or non-identical orders on network. The observable variables of system (1) and system (2) are Q_ix_i and S_ix_{N+i}, respectively. Then the ith error system is

$$e_i(t) = Q_ix_i - S_ix_{N+i}, \quad i = 1,2,...,N, \tag{3}$$

where Q_i and S_i stand for the previous mentioned matrixes, $h_i \leqslant \min(m_i, n_i)$ and each S_i exists the inverse matrix (or the right inverse matrix). It is easy to know that $e_i \in R^{h_i}$ $(i = 1,2,...,N)$.

Let

$$u_i = -f_{N+i}(x_{N+i}) - F_{N+i}(x_{N+i})\hat{P}_{N+i} - \sum_{k=1}^{N} A_{N+i,N+k}x_{N+k} + S_i^{-1}\left(Q_i\left(f_i(x_i) + F_i(x_i)\hat{P}_i + \sum_{k=1}^{N} A_{ik}x_k\right) + \Omega_ie_i\right), \tag{4}$$

where Ω_i is the h_i -order positive definite matrix, $\hat{P}_i$ and $\hat{P}_{N+i}$ are the estimated parameters. Further, the ith error system is

$$\dot{e}_i(t) = Q_i\dot{x}_i - S_i\dot{x}_{N+i} = -Q_iF_i(x_i)\tilde{P}_i + S_iF_{N+i}(x_{N+i})\tilde{P}_{N+i} - \Omega_ie_i, \tag{5}$$

where $\tilde{P}_i = \hat{P}_i - P_i$ and $\tilde{P}_{N+i} = \hat{P}_{N+i} - P_{N+i} (i = 1,2,...,N)$. For convenience, we introduce the following symbols,

$$\begin{aligned}
&x(t) = (x_1^T(t), x_2^T(t), \ldots, x_N^T(t))^T, \quad y(t) = (x_{N+1}^T(t), x_{N+2}^T(t), \ldots, x_{2N}^T(t))^T,\\
&e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T, \quad Q = diag\{Q_1, Q_2, \ldots, Q_N\}, \quad S = diag\{S_1, S_2, \ldots, S_N\},\\
&S^{-1} = diag\{S_1^{-1}, S_2^{-1}, \ldots, S_N^{-1}\}, \quad P_A = \{P_1, P_2, \ldots, P_N\}, \quad P_B = \{P_{N+1}, P_{N+2}, \ldots, P_{2N}\},\\
&f_A = diag\left\{f_1(x_1) + \sum_{k=1}^{N} A_{1k}x_k, f_2(x_2) + \sum_{k=1}^{N} A_{2k}x_k, \ldots, f_N(x_N) + \sum_{k=1}^{N} A_{Nk}x_k\right\},\\
&f_B = diag\left\{f_{N+1}(x_{N+1}) + \sum_{k=1}^{N} A_{N+1,N+k}x_k, f_{N+2}(x_{N+2}) + \sum_{k=1}^{N} A_{N+2,N+k}x_{N+k}, \ldots, f_{2N}(x_{2N}) + \sum_{k=1}^{N} A_{2N,N+k}x_{N+k}\right\},\\
&F_A = diag\{F_1(x_1), F_2(x_2), \ldots, F_N(x_N)\}, \quad F_B = diag\{F_{N+1}(x_{N+1}), F_{N+2}(x_{N+2}), \ldots, F_{2N}(x_{2N})\},\\
&\Omega = diag\{\Omega_1, \Omega_2, \ldots, \Omega_N\}, \quad \tilde{P}_A = (\tilde{P}_1, \tilde{P}_2, \ldots, \tilde{P}_N)^T, \quad \tilde{P}_B = (\tilde{P}_{N+1}, \tilde{P}_{N+2}, \ldots, \tilde{P}_{2N})^T
\end{aligned}$$